Cheap Talk to Multiple Receivers

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Abstract

We study a cheap-talk game with a single sender facing multiple receivers whose actions influence each other's payoff. Even if the sender aims at maximising the expected aggregate payoff of receivers, externalities between receivers result in interest misalignment between sender and receivers and noisy communication. We find conditions such that two-steps partitional equilibria exist, and find that the equilibrium threshold is always higher than the equilibrium threshold. The sender can increase his payoff by delegating communication to an agent with a different objective function than his own. We apply the model to banking markets with externalities in production and to Cournot competition with pecuniary externalities. In the banking application, we identify which fundamentals are decisive for the existence of two-steps partitional equilibria and find that welfare is higher if communication is left to a central bank that places more weight on generating short-term returns than consumers do. For Cournot competition, we show that total welfare can be increased by delegating communication with the firms to an agent who puts more weight on firms than on consumers.

Keywords Bayesian Games · Strategic Communication · Production Externality · Bank Liquidity Creation· Cournot competition

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1 Introduction

In many economic contexts, an outside agent faces a number of agents interacting on a market and is endowed with superior information concerning the economic conditions under which they interact. This information being private and payoff relevant to the latter, a natural scenario is that the outside agent engages in communication with these, for example through public announcements. Prominent examples include economic forecasts by public institutions such as Central Banks, Finance ministries, etc. In such contexts, the outside agent's objectives may however not be perfectly aligned with those of the interacting agents. The government is typically interested in maximising total welfare, the central bank may be keen on keeping inflation in check. Misalignment of interest will make perfect information transmission impossible if communication is Cheap Talk (i.e. messages are costless and unverifiable). The idea being that if believed to always truthfully share its information, the outside agent would want to occasionally lie to bend outcomes in their preferred direction. What information can be revealed, and what are the welfare consequences?

Formally, in the game that we study, a privately informed sender communicates via public cheap talk to multiple receivers. The payoff of the sender and of each receiver depends on the state of the world as well as on the profile of actions chosen by all market players. Market participants thus exert externalities on each other. We consider a sender who is interested in maximising aggregate welfare (we allow for different variants of this). Note that in the presence of externalities, for a known state, strategies of players are inefficient, i.e. market participants do not generate an outcome that maximises total welfare. The sender's communication incentives can thus be separated into two parts. A first concern is to reduce market participants' uncertainty about the state, which everything else equal improves their payoffs. A second concern is to induce the receivers to act in a way that is more efficient given the state. This second concern makes full information revelation impossible to achieve in equilibrium by creating a lying incentive. We provide a simple condition under which partitional equilibria with two intervals exist. Moreover, we show that the sender may benefit from delegating the task of communicating to a different agent whose objective function is biased away from his own total welfare objective, for example by attributing a larger weight to a subgroup of agents.

We then consider applications to two canonical economic models of markets, a model of the market for bank deposits with unknown investment productivity and a model of Cournot competition with unknown demand. In both models, market participants exert an externality on each other; a production externality in the banking model and a pecuniary externality in the Cournot model.

The banking model of Diamond and Dybvig (1983) with uncertainty about the productivity of long-term bank assets is the first application. Consumers want to provide for contingencies that require sudden expenditures prior to the realization of any returns on long-term assets. Banks insure consumers against such idiosyncratic risk. To do so, banks pool consumers' savings and allocate them to risk-free storage and long-term investments. Storage will be given to consumers with early consumption needs and the returns on long-term assets will be shared among consumers who remain patient. As standard, banks are considered to create more liquidity the larger the payout to impatient consumers. There are two frictions. First, the returns on long-term bank assets depend on the aggregate state of the economy. At the time of making their portfolio decisions, banks and consumers know only the distribution of the state. This uncertainty resolves only after consumers learn their own expenditure needs, exposing consumers without early expenditure needs to the risk associated with the long-term assets. Second, there are externalities in production. Specifically, the returns on long-term bank assets are increasing in the aggregate investments in those assets by banks. Hence, banks tend to invest too little in long-term assets and, accordingly, offer to pay too much to impatient consumers. We introduce a central bank that aims at maximising ex ante total welfare and possesses superior information about the future state of the economy. By sharing its information, the central bank reduces uncertainty for banks and consumers. Doing so would be welfare improving if it were not for the externality. As uncertainty about the economy induces banks to be more cautious, and thus curbs their creation of liquidity, a central bank might not find it optimal for banks to be perfectly informed. We show that under standard assumptions, the degree of consumers' risk aversion and the extent to which the productivity of a bank's investments depends on the aggregate state and on aggregate investment are key determinants of the information content of equilibrium central bank communications. For a class of parameters, it is welfare-improving to delegate communication to an agent who puts a larger weight on impatient consumers, and thus prefers more liquidity creation.

Our second application is an oligopolistic market with n firms competing in quantities and facing uncertainty about (linear) demand. We first find that there generically exists no fully revealing equilibrium given that the sender maximises total welfare. A two intervals partitional equilibrium exists only if there are enough firms or the (constant) marginal cost is in an intermediate range. The equilibrium welfare of both firms and consumers is shown to depend exclusively on the informativeness of equilibrium communication, measured in terms of the variance of the conditional expectation of the state induced by equilibrium. If (and only if) the sender's objective function puts weight $\frac{n}{2n-1}$ on firm profits, so that it is slightly tilted in favour of firms, any partition with equally sized intervals constitutes an equilibrium, including the limit case of perfect communication. Generally, a planner interested in maximising total welfare would strictly benefit from delegating communication to any informed party who weighs firm profits between one half and $\frac{n}{2n-1}$.

Literature review: Cheap talk games go back to Crawford and Sobel (1982), and we add to the branch in the literature that considers multiple players. Farrell and Gibbons (1989) introduce cheap talk games with two heterogeneous receivers whose payoffs are not mutually dependent and the conflict of interest between sender and receivers is exogenous. Moreover, the sender can discriminate receivers by deciding to send information in private. There, the existence of multiple receivers affects information transmission as it changes the sender's strategy (for experimental evidence see Battaglini and Makarov, 2014). Others study communication games with multiple informed senders but a single receiver (e.g. Austen-Smith, 1993; Morgan and Stocken, 2008; Li, 2010; Galeotti et al., 2013). We consider games with a single sender and multiple receivers whose payoff functions are symmetric, identical and include payoff externalities. While the sender can only address all receivers at once and there is no private messaging, the conflict of interest between sender and receivers is endogenous as it depends on the extent of the externality.

Secondly, with our application we contribute to the literature on communication strategies as tools for economic policy, e. g. particular monetary policy. Stein (1989) and Bassetto (2019) are among the very few applications of cheap talk to central banking. However, like the companion body of research on central bank communication, they are concerned with the efficacy of monetary policy in an inflation-targeting policy regime plagued with time-inconsistency problems (e.g. Woodford, 2005; Blinder et al., 2008). This literature considers private agents to respond to central bank communication primarily because they anticipate a change in policy, not because they anticipate a change in the economic environment. However, Nakamura and Steinsson (2018) provide empirical evidence that following Federal Open Market Committee announcements, private agents update their beliefs about economic fundamentals too. Similar evidence exists for the UK (Hansen et al., 2019) and the Euro area (Kerssenfischer, 2019). We provide a formal framework to analyze central bank communication where the aim is to inform about the *state of the economy* independently from any considerations about the *future path of monetary policy*.

Another strand in the existing literature on central bank communication is founded in the theory of global games (Carlsson and van Damme, 1993; Morris and Shin, 2002). There, strategic complementarities exist in games among private agents with heterogenous private information. Central bank communication can serve as a coordination device (Morris and Shin, 2008). But if the desire to coordinate is sufficiently strong, private agents will under-value their own information (Morris and Shin, 2018). This literature focuses on the informational efficiency of asset prices and studies the cost and benefit of transparency in central bank communications (see also Lindner, 2006; Ehrmann et al., 2019; Baeriswyl et al., 2020). Like in the global games approach, we consider central bank communication disconnected from policy instruments, making both approaches particularly relevant when providing economic forecasts is considered more important than policy announcements. The cheap talk framework in the present paper differs from global games in that the focus is on *credibility* of communications rather than their *transparency*.

Thirdly, with our application to banking we draw upon the literature starting with Diamond and Dybvig (1983) on banks as creators of liquidity, in particular on their role for aggregate investment and capital formation (e.g. Cooper and Ross, 1998). There is ample, international evidence for investment externalities (e.g. Conley and Dupor, 2003; Harrison, 2003; Diewert et al., 2011). Such externalities are central in models of endogenous growth with liquidity providing banks, where AK-type aggregate production functions are deployed (e.g. Bencivenga and Smith, 1991; Ennis and Keister, 2003; Fecht et al., 2008). However, in those models the externality is not studied as a cause for any inefficiency in its own right, and central bank communication plays no role. We elaborate how investment externalities distort banks' portfolio choices, and demonstrate that central bank communication can mitigate these inefficiencies, but is never a remedy. There is a large, complementary body of research analyzing stability and fragility of banking based on Diamond and Dybvig (1983); see, e.g., Allen and Gale (2000); Peck and Shell (2003); Goldstein and Pauzner (2005). Stability and fragility are not the focus of the present paper.

Our analysis of Cournot markets complements an existing literature on the impact of information about demand or costs on Cournot equilibrium outcomes. The most related strand of this literature studies information provision by an informed outside agent to Cournot players. Eliaz and Forges (2015) consider a planner who observes realised demand and can commit to a disclosure rule which involves communicating privately with Cournot duopolists, the planner's objective being to maximise total profits. The optimal policy is to fully disclose to one firm and disclose no information to the other. If the planner can only use public messages, the optimal policy is to fully inform players. Ghosh and Liu (2020) consider the case where the planner has no commitment and simply engages in strategic ex post disclosure of verifiable signals. In all equilibria, the planner discloses all information. If one allows for private communication, the optimal solution identified by Eliaz and Forges (2015) can be implemented. Kastl et al. (2018) studies an outside agent who sells verifiable information to individual firms playing a Cournot game. They find that the willingness to pay for news may be higher for more imprecise information. The underlying mechanism resides in the price effect of information: more precise increases production on average, causing a lower equilibrium price.

Another strand of the literature studies exogenous information structures. Myatt and Wallace (2015) study a differentiated product oligopoly where each producer obtains multiple signals which can be more or less public (correlated with others' signals). From firms' perspective, information is overused, and excessive weight is placed on public information; from consumers' perspective, information is underused and excessive weight is placed on private information. Welfare would be enhanced by strengthening the use of information but increasing the weight put on private signals.

Bergemann and Morris (2013) studies a general class of games with quadratic payoffs and normally distributed uncertainty about a payoff relevant state, of which Cournot competition with uncertain demand or costs is a special case. Two equilibrium concepts are invoked. In Bayes Correlated equilibrium, agents receive a recommended action from a mediator who observes the state. In Bayes Nash equilibrium, agents receive informative signals (one public and one private) and choose their optimal action in response to these. For every Bayes Correlated equilibrium, there is an information structure for which the implied Bayes Nash equilibrium induces the same action-state distribution. The approach provides a methodology to identify optimal information structures and policies. The authors reconsider the issue of information sharing by firms (see next paragraph). They show that if the price elasticity of demand is high enough, the optimal information policy (maximising total firm profits) centralises firms' private signals and generates a noisy public signal of the average private signal of agents, thereby optimally trading off the advantage of better information about the state against the downside of correlation in individual quantity choices. For more on information design approaches, see Mathevet et al. (2020), which takes a general approach to information design within the context of games.

Finally, there is also a literature considering the sharing of information on demand or costs held in a decentralised fashion by market participants (Clarke (1983); Vives (1984)). Clarke (1983) finds that firms do not share any information about demand under constant marginal costs, but it was subsequently shown that the result can be reversed under convex costs. Goltsman and Pavlov (2014) consider one shot cheap talk communication by firms. They find that communication through a third party can be informative when informative cheap is impossible. They construct a simple mediated mechanism that interim Pareto dominates the case where firms act solely on the basis of their private information.

The remainder of the paper is organized as follows. In Section 2 we present and study a general cheap-talk game with multiple receivers whose payoffs are mutually dependent. In Section 3 we apply this framework to central bank communication and discuss implications for its credibility. In Section 4 we apply the framework to finance ministry communication and discuss implications for its credibility. The final section concludes.

2 A general model

In the present section we set up a general cheap-talk game with a sender and multiple receivers. There are results on: existence of partially revealing signaling equilibria; comparison of equilibrium thresholds with optimal thresholds; and, whether the sender can improve welfare by delegating to another agent with different preferences.

2.1 Setup and definitions

There is a state s which is drawn from a uniform distribution on the state space $S = [0,1]$. There is sender $\mathscr{C}$ who privately observes the state and there are n market players who do not observe the state. After observing the state, $\mathscr{C}$ sends a payoff-irrelevant message from the message set $M = S$. After observing the message sent by $\mathscr{C}$, market players simultaneously pick actions from the action space $A =]0,1[$. Payoffs to both, $\mathscr{C}$ and market players, depend exclusively on the actions picked by market players as well as the state.

Payoff functions: Market players' payoff functions are symmetric and identical $u_i : A^n \times S \to \mathbb{R}$. The payoff of player i depends on the state and the full profile of strategies across market players. Holding fixed the profile of strategies played by all market participants but i and j, players i and j get the same payoff provided they play identical strategies. Every market player aims at maximising her individual payoff. Payoff functions of market players are twice differentiable and supposed to satisfy the following assumptions:

(A.1) $\lim_{a\to 0} \dfrac{\partial u_i(a,\dots,a,s)}{\partial a_i} > 0 > \lim_{a\to 1} \dfrac{\partial u_i(a,\dots,a,s)}{\partial a_i}$ and $\dfrac{\partial^2 u_i(a_1,\dots,a_n,s)}{\partial a_i \partial a_i} < 0$.

(A.2) $\sum_j \dfrac{\partial^2 u_i(a,\dots,a,s)}{\partial a_i \partial a_j} < 0$ and $\dfrac{\partial^2 u_i(a,\dots,a,s)}{\partial a_i \partial s} > 0$.

Assumption (A.1) states that on the diagonal, where all market players play identical strategies, the sign of the derivative of the payoff of agent i with respect to her own strategy changes from positive for low a's to negative for high a's. Assumption (A.2) states that the derivative of the payoff of player i is differentiable decreasing in uniform changes of strategies of all players and differentiable increasing in the state.

The sender $\mathscr{C}$ aims at maximising a symmetric payoff function $u : A^n \times S \to \mathbb{R}$, so for every permutation μ of the set of players $\{1,\dots,n\}$ and all states s,

$$u(a_{\mu(1)},\dots,a_{\mu(n)},s) \;=\; u(a_1,\dots,a_n,s).$$

The payoff function for $\mathscr{C}$ satisfies the following assumptions:

(A.3) $\dfrac{\partial u(a,\ldots,a,s)}{\partial a_i} > \dfrac{\partial u_i(a,\ldots,a,s)}{\partial a_i}$.

(A.4) $\lim_{a\to 0} \dfrac{\partial u(a,\ldots,a,s)}{\partial a_i} > 0 > \lim_{a\to 1} \dfrac{\partial u(a,\ldots,a,s)}{\partial a_i}$.

(A.5) $\sum_{i,j} b_i b_j \dfrac{\partial^2 u(a_1,\ldots,a_n,s)}{\partial a_i \partial a_j} < 0$ for all $b \neq 0$ and $\dfrac{\partial^2 u(a,\ldots,a,s)}{\partial a_i \partial s} > 0$.

Assumption (A.3) states that the derivative of the payoff of $\mathscr{C}$ with respect to the strategy of player i is larger than the derivative of the payoff of agent i with respect to her own strategy. Assumption (A.4) states that the sign of the derivative of $\mathscr{C}$'s payoff with respect to the strategy of a player changes from positive to negative on the diagonal. Assumption (A.5) states that the payoff of $\mathscr{C}$ is differentiable strictly concave and that the derivative with respect to the strategy of a player is differentiable increasing in the state.

One example of u is

$$u(a_1,\ldots,a_n,s) \;=\; \sum_i u_i(a_1,\ldots,a_n,s)$$

which applies to a market where every player is a bank acting on behalf of consumers and the payoff for every consumer is her expected utility, and the planner is an agency supporting the consumers and consequently aiming at maximising aggregate consumer welfare. We will use this specification in Section 3.

Another example of u is

$$u(a_1,\ldots,a_n,s) \;=\; \sum_i u_i(a_1,\ldots,a_n,s) + g(a_1,\ldots,a_n,s),$$

where g is symmetric. This example applies to a market where every player is a firm and the planner is a government agency aiming at maximising the sum of total producer surplus $\sum_i u_i(\cdot)$ and total consumer surplus $g(\cdot)$. We will use this specification in Section 4.

Communication strategies: A *communication strategy* is a measurable function from the state space to the message space $\sigma : S \to M$. The *fully revealing signaling strategy*, denoted σ_{FR}, is the identity function, i.e. satisfies $\sigma_{\text{FR}}(s) = s$ for all s. Without loss of generality, a *partitional communication strategy* is defined by a finite set of thresholds $\{s_0, s_1, \ldots, s_N\} \in [0,1]$, with $0 = s_0 < \ldots < s_N = 1$, such that for every $r \in \{1, \ldots, N\}$ and all $s \in]s_{r-1}, s_r]$, $\sigma(s) = s_r$. A partitional signaling strategy thus partitions the set of states into finitely many open intervals and market players simply learn in which of these intervals the state is located. A partitional signaling strategy is *partially revealing* if it features at least two intervals (i.e. $N \geq 2$) and it is *non-revealing* if it features only one interval. We denote by σ_{NR} the non-revealing strategy summarized by $s_1 = 1$, in which $\mathscr{C}$ always sends message 1. Concerning out-of-equilibrium beliefs in a given equilibrium featuring the partitional strategy $\{s_0, \ldots, s_N\}$, we assume that a message $m \in]s_{r-1}, s_r[$ gives rise to the same belief as s_r.

Market player strategies: A pure *strategy* for market player i is a map $\alpha_i : M \to A$, which specifies a pure action chosen conditional on each $m \in M$, denoted by $a_i(m)$. A *strategy profile* is a list of strategies $\alpha = (\alpha_1, \ldots, \alpha_m)$. In a putative equilibrium featuring the partitional strategy pinned down by $\{s_0, s_1, \ldots, s_N\}$, let $U_i(a_1, \ldots, a_n, s_{r-1}, s_r)$ be the expected utility for player i of action profile $(a_1, \ldots, a_n)$ conditional on observing the signal s_r. We have:

$$U_i(a_1, \ldots, a_n, s_{r-1}, s_r) \;=\; \frac{1}{s_r - s_{r-1}} \int_{s_{r-1}}^{s_r} u_i(a_1, \ldots, a_n, s)\, \partial s.$$

Equilibrium definition: Our equilibrium concept is perfect Bayesian equilibrium, which is given by a profile of strategies such that strategies are sequentially rational given beliefs, while beliefs on the equilibrium path are derived by Bayes'rule. Sequential rationality of market players' strategies requires that in each market subgame, as determined by the publicly observed message sent by the sender, the profile of market players' strategies is a Nash equilibrium. Sequential rationality of $\mathscr{C}$'s strategy requires that whatever the realised state, $\mathscr{C}$ has no strict incentive to deviate from his equilibrium message, correctly anticipating the action profile that results from every message in the message set.

Definition 1 *A **partitional equilibrium** is given by a partitional communication strategy σ pinned down by $\{s_0, s_1, \dots, s_N\}$ and a profile of market player strategies $\alpha = (\alpha_1, \dots, \alpha_m)$ such that:*

- *For every $r \in \{1, \dots, N\}$, if $s \in]s_{r-1}, s_r]$, then for every $m' \neq s$:*

$$u(a_1(s_r), \dots, a_n(s_r), s) \geq u(a_1(m'), \dots, a_n(m'), s).$$

- *For every message $m \in M$, the action profile $(a_1(m), \dots, a_n(m))$ is a **Nash** equilibrium given signal m: $\forall r \in \{1, \dots, N\}$, $m \in]s_{r-1}, s_r]$, $a'_i \neq a_i(m)$:*

$$U_i(a_1(m), \dots, a_n(m), s_r, s_{r+1}) \geq U_i(a_1(m), \dots, a_{i-1}(m), a'_i, a_{i+1}(m), \dots, a_n(m), s_r, s_{r+1}).$$

2.2 Equilibrium

We start by studying equilibria of the market subgame. A Nash equilibrium α is said to be symmetric if $\alpha_i = \alpha_j$ for every pair of players i and j.

Lemma 1 *Let σ_{FR} be the perfectly revealing signaling strategy, $\sigma(s) = s$ for all s.*

- *For all s, there is a unique symmetric Nash equilibrium in which every player plays $a^{\sigma_{FR}}(s)$.*
- *For all $s < t$, $a^{\sigma_{FR}}(s) < a^{\sigma_{FR}}(t)$.*

Proof: According to the first part of (A.1), there is a such that $\partial u_i(a, \dots, a, s)/\partial a_i = 0$. According to the second part of (A.1), u_i is strictly concave in a_i. Therefore $(a, \dots, a)$ is a symmetric Nash equilibrium. At a symmetric Nash equilibrium with $a_i = a$ for every i,

$$\frac{\partial u_i(a,\ldots,a,s)}{\partial a_i} \;=\; 0. \tag{1}$$

According to (A.2), the derivative of the left side of Equation (1) with respect to a is negative,

$$\sum_j \frac{\partial^2 u_i(a,\ldots,a,s)}{\partial a_i \partial a_j} \;<\; 0.$$

Hence, there is a unique symmetric Nash equilibrium $a^{\sigma_{\mathrm{FR}}}(s)$ for all s.

The derivative of the left side of Equation (1) with respect to s is positive according to (A.2). Therefore $\mathrm{d}a^{\sigma_{\mathrm{FR}}}(s)/\mathrm{d}s > 0$ according to the implicit function theorem. □

Lemma 2 *Let σ be a partitional communication strategy with threshold profile $\{s_0, s_1, \ldots, s_N\}$. Then every message $m \in M$ induces a unique symmetric Nash equilibrium in which every player plays $a^{\sigma}(m)$.*

Proof: According to the first part of (A.1), for every r there is a such that $\partial U_i(a,\ldots,a,s_r,s_{r+1})/\partial a_i = 0$. According to the second part of (A.1), for every r, U_i is strictly concave in a_i. Therefore $(a,\ldots,a)$ is a symmetric Nash equilibrium given message s_r, which implies that $s \in\,]s_{r-1}, s_r]$. At a symmetric Nash equilibrium, for every $r \in \{1,\ldots,n\}$ and all $s \in]s_r, s_{r+1}[$,

$$\frac{\partial U_i(a,\ldots,a,s_r,s_{r+1})}{\partial a_i} \;=\; 0. \tag{2}$$

According to (A.2), the derivative of Equation (2) with respect to a is negative,

$$\sum_j \frac{\partial^2 U_i(a,\ldots,a,s_r,s_{r+1})}{\partial a_i \partial a_j} \;<\; 0.$$

Hence there is a unique symmetric Nash equilibrium $a^{\sigma}(m)$ for all messages $m \in M$. □

For all partially revealing partitional communication strategies, the equilibrium action is increasing in the partition interval implied by S's message. Furthermore, the action is located between the Nash equilibrium actions corresponding to the endpoints of the interval implied by the sent message. Finally, the action is increasing in the boundaries of the interval.The above is summarised in our next Lemma.

Lemma 3 *Let σ be a partitional communication strategy with threshold profile $\{s_0, s_1, \ldots, s_N\}$.*

- *For every $r \in \{1, \ldots, N-1\}$, $a^{\sigma_{FR}}(s_{r+1}) > a^{\sigma}(s_r) > a^{\sigma_{FR}}(s_r)$.*
- *For every $r \in \{1, \ldots, N-1\}$, $a^{\sigma}(s_r) < a^{\sigma}(s_{r+1})$.*
- *For every $r \in \{1, \ldots, N-1\}$,*

$$\frac{\partial a^{\sigma}(s_r)}{\partial s_r}, \frac{\partial a^{\sigma}(s_r)}{\partial s_{r+1}} \; > \; 0.$$

Proof: According Lemma 2 there is a unique symmetric Nash equilibrium for σ. According to (A.2) , the left side of Equation (2) is negative for $a = a^{\sigma_{\text{FR}}}(s_{r+1})$ and positive for $a = a^{\sigma_{\text{FR}}}(s_r)$. Therefore $a^{\sigma_{\text{FR}}}(s_{r+1}) > a^{\sigma}(s_r) > a^{\sigma_{\text{FR}}}(s_r)$ for every r.

Since $a^{\sigma_{\text{FR}}}(s_{r+1}) > a^{\sigma}(s_r) > a^{\sigma_{\text{FR}}}(s_r)$ for every $r \leq N-1$, $a^{\sigma}(s_r) < a^{\sigma}(s_{r+1})$ for all $r < N-1$.

Suppose σ is a partitional strategy with threshold profile $\{s_0, s_1, \ldots, s_N\}$ and $a^{\sigma}(s_r)$ is a symmetric Nash equilibrium. Then,

$$\sum_j \frac{\partial^2 U_i(a, \ldots, a, s_r, s_{r+1})}{\partial a_i \partial a_j} \; < \; 0$$

and since $a^{\sigma_{\text{FR}}}(s_r) < a^{\sigma}(s_r) < a^{\sigma_{\text{FR}}}(s_{r+1})$ for every r,

$$\begin{aligned} \frac{\partial^2 U_i(a^{\sigma}(s_r),\ldots,a^{\sigma}(s_r),s_r,s_{r+1})}{\partial a_i \partial s_r} &= -\frac{\partial u_i(a^{\sigma}(s_r),\ldots,a^{\sigma}(s_r),s_r)}{\partial a_i} > 0 \\ \frac{\partial^2 U_i(a^{\sigma}(s_r),\ldots,a^{\sigma}(s_r),s_r,s_{r+1})}{\partial a_i \partial s_{r+1}} &= \frac{\partial u_i(a^{\sigma}(s_r),\ldots,a^{\sigma}(s_r),s_{r+1})}{\partial a_i} > 0. \end{aligned}$$

Hence, $\partial a^{\sigma}(s_r)/\partial s_r, \partial a^{\sigma}(s_r)/\partial s_{r+1} > 0$ according to the implicit function theorem. □

Before stating our results concerning the set of partitional equilibria, we add a remark concerning the optimal action profile for any given s from $\mathscr{C}$'s perspective. As stated next, there is a unique symmetric solution to the planner problem for all states, the solution is increasing in the state and larger than the equilibrium strategy in all states.

Lemma 4 *Consider the decision problem* (A,S,u).

- *For all s, there is a unique symmetric solution to the planner problem* $a^P(s)$.
- *For all* $s < t$, $a^P(s) < a^P(t)$.
- *For all s,* $a^P(s) > a^{\sigma_{FR}}(s)$.

Proof: The proofs of the three postulates follow the proof of Lemma 1 using (A.4) instead of (A.1) and (A.5) instead of (A.2). □

We may now state our results concerning the set of partitional equilibria. Note first that trivially there exists no fully revealing signaling equilibrium.

Theorem 1 *If* σ *is a signaling equilibrium, then* $\sigma \neq \sigma_{FR}$.

Proof: For all $s \in S$, $a^{\sigma_{\text{FR}}}(s) < a^P(s)$ according to Lemma 4 and the derivatives of $a^{\sigma_{\text{FR}}}$ and a^P with respect to s are positive. Moreover, the derivative of $u(a^{\sigma_{\text{FR}}}(t),\ldots,a^{\sigma_{\text{FR}}}(t),s)$ with respect to t is positive at $t = s$ according to Assumption (A.4) and Lemma 4. Therefore, for all $s < 1$, $\mathscr{C}$ can increase his payoff by increasing the message. □

Turning to partially revealing signaling equilibria, we focus on partitional communication σ with two intervals for simplicity.

Theorem 2 *For state $s = 0$ suppose the utility of $\mathscr{C}$ is higher at the Nash equilibrium $a^{\sigma_{FR}}(0)$ under the fully revealing communication strategy than at the Nash equilibrium $a^{\sigma_{NR}}(1)$ under the non-revealing communication strategy,*

$$u(a^{\sigma_{FR}}(0),\ldots,a^{\sigma_{FR}}(0),0) \;>\; u(a^{\sigma_{NR}}(1),\ldots,a^{\sigma_{NR}}(1),0).$$

Then there is a partially revealing signaling equilibrium.

Proof: Consider a partitional communication strategy σ with pinned down by threshold $s_1 \in (0,1)$. Suppose the state is s_1. Then for $m = s_1$, the utility of the planner is $u(a^{\sigma}(s_1),\ldots,a^{\sigma}(s_1),s_1)$ with

$$\begin{aligned}
\lim_{s_1\to 0} u(a^{\sigma}(s_1),\ldots,1^{\sigma}(s_1),s_1) &= u(a^{\sigma_{\mathrm{FR}}}(0),\ldots,a^{\sigma_{\mathrm{FR}}}(0),0)\\
\lim_{s_1\to 1} u(a^{\sigma}(s_1),\ldots,a^{\sigma}(s_1),s_1) &= u(a^{\sigma_{\mathrm{NR}}}(1),\ldots,a^{\sigma_{\mathrm{NR}}}(1),1).
\end{aligned}$$

Given $m = s_2 = 1$, the utility of the planner is $u(\alpha^{\sigma}(t),\ldots,\alpha^{\sigma}(t),s_1)$ with

$$\begin{aligned}
\lim_{s_1\to 0} u(a^{\sigma}(1),\ldots,a^{\sigma}(1),s_1) &= u(a^{\sigma_{\mathrm{NR}}}(1),\ldots,a^{\sigma_{\mathrm{NR}}}(1),0)\\
\lim_{s_1\to 1} u(a^{\sigma}(1),\ldots,a^{\sigma}(1),s_1) &= u(a^{\sigma_{\mathrm{FR}}}(1),\ldots,a^{\sigma_{\mathrm{FR}}}(1),1).
\end{aligned}$$

By assumption, $u(a^{\sigma_{\mathrm{FR}}}(0),\ldots,a^{\sigma_{\mathrm{FR}}}(0),0) > u(a^{\sigma_{\mathrm{NR}}}(1),\ldots,a^{\sigma_{\mathrm{NR}}}(1),0)$. Since $a^{P}(1) > a^{\sigma_{\mathrm{FR}}}(1) > a^{\sigma_{\mathrm{NR}}}(1)$, $u(a^{\sigma_{\mathrm{NR}}}(1),\ldots,a^{\sigma_{\mathrm{NR}}}(1),1) < u(a^{\sigma_{\mathrm{FR}}}(1),\ldots,a^{\sigma_{\mathrm{FR}}}(1),1)$. Therefore there is $s_1 \in]0,1[$ such that

$$u(a^{\sigma}(s_1),\ldots,a^{\sigma}(s_1),s_1) \;=\; u(a^{\sigma}(1),\ldots,a^{\sigma}(1),s_1).$$

Consequently, there exists a partially revealing signaling equilibrium. □

The above argument relies on a standard intermediate value argument, which guarantees the existence of a state at which the required indifference condition holds.

At a partially revealing signaling equilibrium with threshold $s_1 \in (0,1)$, if the state is s_1, then the planner is indifferent between sending the signal associated with states below s_1 and sending the signal associated with states above s_1. Since the perfectly revealing signaling strategy is not a signaling equilibrium according to Theorem 1, the actions induced by the signal associated with states below (above) s_1 are lower (higher) than the optimal action.

2.3 Information and welfare

Consider a partitional partially revealing communication strategy σ with two intervals pinned down by $s_1 = \tau$, where $\tau \in]0,1[$. Let $a_L^\tau = a^\sigma(\tau)$ and $a_H^\tau = a^\sigma(1)$ be the respective Nash equilibrium strategies associated with equilibrium messages τ and 1. Let $U^\tau : [0,1] \to \mathbb{R}$ be the expected payoff for $\mathscr{C}$ as a function of the threshold

$$U^\tau = \int_0^\tau u(a_L^\tau, \ldots, a_L^\tau, s)\, ds + \int_\tau^1 u(a_H^\tau, \ldots, a_L^\tau, s)\, ds$$

Then an optimal threshold for $\mathscr{C}$ for a partitional partially revealing communication strategy is a solution to $\max_\tau U^\tau$. For simplicity, assume U^τ is unimodal so there is a unique optimal threshold τ^*.

Theorem 3 *The optimal threshold for $\mathscr{C}$ is larger than the equilibrium threshold $\tau^* > s_1$.*

Proof: Consider a partitional partially revealing communication strategy σ with two intervals pinned down by $s_1 = \tau$, where $\tau \in]0,1[$. Let $a_L^\tau = a^\sigma(\tau)$ and $a_H^\tau = a^\sigma(1)$ be the respective Nash equilibrium strategies associated with equilibrium messages τ and 1. Then the derivative of U^τ is

$$\begin{aligned} U^{\tau\prime} &= u(a_L^\tau,\ldots,a_L^\tau,\tau)+\int_0^\tau \sum_j \frac{\partial u(a_L^\tau,\ldots,a_L^\tau,s)}{\partial a_j}\,\frac{da_L^\tau}{d\tau}\,ds \\ &\quad -u(a_H^\tau,\ldots,a_H^\tau,\tau)+\int_\tau^1 \sum_j \frac{\partial u(a_H^\tau,\ldots,a_H^\tau,s)}{\partial a_j}\,\frac{da_H^\tau}{d\tau}\,ds. \end{aligned}$$

Since σ is an equilibrium partitional strategy, for $\tau = s_1$ we have $u(a_L^\tau,\ldots,a_L^\tau,\tau) = u(a_H^\tau,\ldots,a_H^\tau,\tau)$ and

$$\int_0^\tau \frac{\partial u_i(a_L^\tau,\ldots,a_L^\tau,s)}{\partial a_i}\,ds \;=\; \int_\tau^1 \frac{\partial u_i(a_H^\tau,\ldots,a_H^\tau,s)}{\partial a_i}\,ds \;=\; 0$$

for every i. Therefore for $\tau = s_1$ the derivative of U^τ is positive

$$\begin{aligned} U^{\tau\prime} &= \int_0^\tau \sum_j \frac{\partial u(a_L^\tau,\ldots,a_L^\tau,s)}{\partial a_j}\,\frac{da_L^\tau}{d\tau}\,ds+\int_\tau^1 \sum_j \frac{\partial u(a_H^\tau,\ldots,a_H^\tau,s)}{\partial a_j}\,\frac{da_H^\tau}{d\tau}\,ds \\ &> \int_0^\tau \sum_j \frac{\partial u_j(a_L^\tau,\ldots,a_L^\tau,s)}{\partial a_j}\,\frac{da_L^\tau}{d\tau}\,ds+\int_\tau^1 \sum_j \frac{\partial u_j(a_H^\tau,\ldots,a_H^\tau,s)}{\partial a_j}\,\frac{da_H^\tau}{d\tau}\,ds \;=\; 0 \end{aligned}$$

because $\partial u(a,\ldots,a,s)/\partial a_i > \partial u_i(a,\ldots,a,s)/\partial a_i$ for every i according to Assumption (A.3) and $da_L^\tau/d\tau, da_H^\tau/d\tau > 0$ according to Lemma 3. Hence, $\tau^* > s_1$ because U^τ is unimodal. □

We add some comments on the comparison of welfare across two scenarios, namely the fully revealing signaling strategy σ_{FR} and, alternatively, the non-revealing signaling strategy σ_{NR}. The expected utility of $\mathscr{C}$ for σ_{FR} is

$$U^{\sigma_{\mathrm{FR}}} \;=\; \int u(a^{\sigma_{\mathrm{FR}}}(s),\ldots,a^{\sigma_{\mathrm{FR}}}(s),s)\,ds$$

while for σ_{NR} it is

$$U^{\sigma_{\mathrm{NR}}} \;=\; \int u(a^{\sigma_{\mathrm{NR}}}(1),\ldots,a^{\sigma_{\mathrm{NR}}}(1),s)\,ds.$$

Since $a^{\sigma_{\text{FR}}}(0) < a^{\sigma_{\text{NR}}}(1)$ according to Lemma 3, if the payoff for actions lower than $a^{\sigma_{\text{NR}}}(1)$ is very low, then $\mathscr{C}$ is better off with σ_{NR} than with σ_{FR}. Let the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) \;=\; \begin{cases} e^{1/x} & \text{for } x < 0 \\ \\ 0 & \text{for } x \geq 0 \end{cases}$$

so $f \in C^\infty(\mathbb{R}, -\mathbb{R}_+)$ with $f'(x) > 0$ for all $x < 0$ and $f''(x) \leq 0$ for $x \geq -1/2$. For $u_\delta : A^m \times S \to \mathbb{R}$ with $\delta \geq 0$ defined by

$$u_\delta(a_1,\ldots,a_n,s) \;=\; u(a_1,\ldots,a_n,s) - \delta \sum_i f\left(\frac{a_i - a^P(0)}{2a^P(0)}\right)$$

if $a^P(s)$ is the unique symmetric solution to the planner problem for payoff function u and state s, then $a^P(s)$ is the unique symmetric solution to the planner problem for payoff function u^δ and state s too because $u_\delta(a_1,\ldots,a_n,s) \leq u(a_1,\ldots,a_n,s)$ for all $(a_1,\ldots,a_n,s) \in a^{\sigma_{\text{FR}}} \times S$ and $u_\delta(a^P(s),\ldots,a^P(s),s) = u(a^P(s),\ldots,a^P(s),s)$ for all $s \in S$. Moreover, there is $\bar{\delta} \geq 0$ such that $U_\delta^{\sigma_{\text{FR}}} < U_\delta^{\sigma_{\text{NR}}}$ for all $\delta > \bar{\delta}$.

2.4 Delegated communication

Could $\mathscr{C}$ be better off by delegating communication to another agent who also observes the state but is endowed with another payoff function? For simplicity, we assume the assumption of Theorem 2 is satisfied, U^τ is unimodal and there is a unique partitional equilibrium with two intervals and focus this equilibrium. As corollaries to Theorem 3 we can characterize the agents to whom $\mathscr{C}$ could delegate to be better off.

Assuming $\mathscr{C}$ aims at maximising the sum of welfare of the market players and some other agents whose welfare depends on the actions taken by the market players, consider the class of payoff functions $u_\lambda : A^n \times S \to \mathbb{R}$ parameterized by $\lambda \in [0,1]$ where

$$u_\lambda(a_1,\ldots,a_n,s) \;=\; (1-\lambda)u(a_1,\ldots,a_n,s) + \lambda \sum_i u_i(a_1,\ldots,a_n,s).$$

Obviously, for $\lambda = 0$ the agent aims at maximising the payoff of $\mathscr{C}$ while for $\lambda > 0$ the agents puts more weight on the welfare of the other agents than $\mathscr{C}$ does. For simplicity assume there is a unique partitional equilibrium with two steps for all λ in some neighbourhood of zero.

Corollary 1 *Suppose that for all a, b with $a < b$ and all s, $u(a, \ldots, a, s) = u(b, \ldots, b, s)$ implies $\sum_i u_i(a, \ldots, a, s) > \sum_i u_i(b, \ldots, b, s)$. Then there is $\bar{\lambda} > 0$ such that $\mathscr{C}$ is better off by delegating communication to another agent with payoff function u_λ for all $\lambda \in]0, \bar{\lambda}[$.*

Proof: As σ is an equilibrium partitional strategy, we have:

$$u_\lambda(a_L^\tau, \ldots, a_L^\tau, \tau) - u_\lambda(a_H^\tau, \ldots, a_H^\tau, \tau) \;=\; \lambda \left(\sum_i u_i(a_L^\tau, \ldots, a_L^\tau, \tau) - \sum_i u_i(a_H^\tau, \ldots, a_H^\tau, \tau) \right)$$

because $u(a_L^\tau, \ldots, a_L^\tau, \tau) = u(a_H^\tau, \ldots, a_H^\tau, \tau)$, so for $\lambda > 0$,

$$u_\lambda(a_L^\tau, \ldots, a_L^\tau, \tau) - u_\lambda(a_H^\tau, \ldots, a_H^\tau, \tau) \;>\; 0.$$

Therefore, it follows from the proof of Theorem 2 that if σ_λ is the unique partitional equilibrium with two steps under a sender with utility function u_λ, then the threshold s_1^λ for $\lambda > 0$ is larger than the threshold s_1. Moreover, the threshold s_1 is continuous in λ, so there is $\bar{\lambda} > 0$ such that $\mathscr{C}$ is better off by delegating to an agent with payoff function u_λ for all $\lambda \in]0, \bar{\lambda}[$ because $\tau^* > s_1$ according to Theorem 3. □

The assumption of Corollary 1 states that if two action profiles, low and high, result in identical payoffs for the planner, then low results in a greater aggregate payoff than high for the players. Therefore, the assumption implies that the action profile maximising the aggregate payoff of players is lower than the action profile maximising $\mathscr{C}$'s payoff. Corollary 1 shows that $\mathscr{C}$ can be better off by delegating signaling to an agent who puts less weight on $\mathscr{C}$'s payoff and more weight on market players' payoffs.

Assuming that $\mathscr{C}$ aims at maximising the sum of market players' utilities, consider the class of payoff functions $u_\lambda : A^n \times S \to \mathbb{R}$ parameterized by $\lambda \in [0,1]$ where

$$u_\lambda(a_1,\ldots,a_n,s) \;=\; (1-\lambda)\sum_i u_i(a_1,\ldots,a_n,s) - \lambda \sum_i a_i.$$

Obviously, for $\lambda = 0$ the agent aims at maximising the payoff of $\mathscr{C}$ while for $\lambda > 0$ there is an additional cost increasing in strategies. For simplicity assume there is a unique partitional equilibrium with two intervals for all λ in some neighbourhood of zero.

Corollary 2 *There is $\bar{\lambda} > 0$ such that $\mathscr{C}$ is better off by delegating to another agent with payoff function u_λ for all $\lambda \in]0,\bar{\lambda}[$.*

Proof: Since $u(a,\ldots,a,s) = \sum_i u_i(a,\ldots,a,s)$ and $\partial u(a,\ldots,a,s)/\partial a_i > \partial u_i(a,\ldots,a,s)/\partial a_i$ according to Assumption (A.3), $\sum_{j\neq i} \partial u_j(a,\ldots,a,s)/\partial a_i > 0$. Therefore, the derivative of U^τ with respect to τ at $\tau = s_1$ is positive,

$$\begin{aligned} U^{\tau\prime} &= \int_0^\tau \sum_i \sum_j \frac{\partial u_i(a_L^\tau,\ldots,a_L^\tau,s)}{\partial a_j} \frac{da_L^\tau}{d\tau}\, ds + \int_\tau^1 \sum_i \sum_j \frac{\partial u_i(a_H^\tau,\ldots,a_H^\tau,s)}{\partial a_j} \frac{da_H^\tau}{d\tau}\, ds \\ &= \int_0^\tau \sum_i \sum_{j\neq i} \frac{\partial u_i(a_L^\tau,\ldots,a_L^\tau,s)}{\partial a_j} \frac{da_L^\tau}{d\tau}\, ds + \int_\tau^1 \sum_i \sum_{j\neq i} \frac{\partial u_i(a_H^\tau,\ldots,a_H^\tau,s)}{\partial a_j} \frac{da_H^\tau}{d\tau}\, ds \;>\; 0, \end{aligned}$$

because $da_L^\tau/d\tau,\, da_H^\tau/d\tau > 0$ according to Lemma 3.

Since σ is an equilibrium partitional strategy, for $\tau = s_1$ we have:

$$u_\lambda(a_L^\tau,\ldots,a_L^\tau,\tau) - u_\lambda(a_H^\tau,\ldots,a_H^\tau,\tau) \;=\; -\lambda n(a_L^\tau - a_H^\tau)$$

because $u_i(a_L^\tau,\ldots,a_L^\tau,\tau) = u_i(a_H^\tau,\ldots,a_H^\tau,\tau)$ for every i, so for $\lambda > 0$,

$$u_\lambda(a_L^\tau,\ldots,a_L^\tau,\tau) - u_\lambda(a_H^\tau,\ldots,a_H^\tau,\tau) \;>\; 0$$

Therefore, it follows from Lemma 3 and the proof of Theorem 2 that if σ_λ is the unique partitional equilibrium with two steps under a sender with utility function u_λ, then the threshold s_1^λ for $\lambda > 0$ is higher than the threshold s_1. Moreover, the threshold s_1^λ is continuous in λ, so there is $\bar{\lambda} > 0$ such that the planner is better off by delegating to an agent with payoff function u_λ for all $\lambda \in]0, \bar{\lambda}[$. □

Corollary 2 shows that $\mathscr{C}$ can increase welfare by delegating to an agent who wants the market players to use lower strategies than $\mathscr{C}$ wants independently of the payoff function of $\mathscr{C}$.

3 Communicating with competitive banks

In the present section we apply the general cheap-talk game to a canonical model of financial intermediation. Our focus is on two-steps partional equilibria. All insights from the general cheap-talk game hold here directly. The purpose of this application is to show that a conflict of interest between the sender and the market players can arise solely from production externalities; to identify parameters relating to preferences and technologies that govern the existence of equilibria featuring a partitional communication strategy; and, to show that delegation can improve welfare even without employing the services of an agent who cares about actions as such or about a combination of the sender's and the market players' payoffs.

3.1 Setup and definitions

We consider a banking market where banks create liquidity on behalf of consumers (as in Diamond and Dybvig, 1983) in an environment with aggregate risk about productivity (as in Allen and Gale, 1998). We make two extensions. First, the returns on productive investments are increasing in aggregate investments (as in the endogenous growth model of

Romer, 1990). Second, there is a benevolent central bank endowed with private information about productivity.

There are three dates $T \in \{0,1,2\}$ with one good at every date. The good can be consumed, stored, and used as input for the production of goods. There is a continuum of consumers $i \in [0,1]$, a continuum of banks $j \in [0,1]$, and a central bank. The state s of the economy is drawn from a uniform distribution on $S = [0,1]$ about which consumers and banks learn at the final date $T = 2$ and the central bank already at date $T = 0$.

All consumers are identical ex-ante and endowed with one unit of the good at date $T = 0$. With probability $t > 0$ a consumer becomes impatient and interested solely in consumption $c_1 > 0$ at date $T = 1$. With probability $1-t > 0$, a consumer becomes patient and interested solely in consumption $c_2 > 0$ date $T = 2$. Consumers learn their types only at date $T = 1$. Even though the law of large numbers does not hold with a continuum of random variables, it is assumed the fraction of impatient consumers is t and the fraction of patient consumers is $1-t$. Indeed, the set of consumers being patient could be determined by distributing consumers uniformly on a circle, making a random draw of a point on the circle and letting consumers close enough to the point be impatient and the rest patient. Let $v : \mathbb{R}_{++} \to \mathbb{R}$ be the Bernoulli utility function of consumers. Thus, consumers value consumption with $v(c_1)$ if impatient and with $v(c_2)$ if patient and their expected utility at $T = 0$ is $tv(c_1) + (1-t)v(c_2)$.

Banks are identical and have access to two different technologies. The first is storage. It can be used at dates $T \in \{0,1\}$ and its returns between subsequent dates are one-for-one. Let the amount stored between dates $T = 0$ and $T = 1$ be x. The second technology is more productive than storage but takes also more time to mature as it transforms input at $T = 0$ into output only at date $T = 2$. The productivity of this technology depends on the aggregate state s. Moreover, there is an externality in production in that the rate of return depends on aggregate investment in production. Specifically, let y_j be the investment by bank j and $Y = \int_0^1 y_k \, \mathrm{d}k$ the aggregate investment in production. Then production by bank

j is $B(Y,s)y_j$. There is Bertrand competition among banks, so profits are competed away in equilibrium because both technologies are linear.

The central bank is interested in maximising the expected utility of consumers. Knowing the state s, it can act as a sender $\mathscr{C}$ and communicate a non-verifiable message about the state to consumers and banks as the market players at no cost before they make their decisions at date $T=0$.

An economy is described by (t,v,B). Economies satisfy the following assumptions:

(B.1) $v \in C^2(\mathbb{R}_{++},\mathbb{R})$ with $v'(c)>0>v''(c)$ and $-cv''(c)/v'(c)>1$ for all c and $\lim_{c\to 0} v'(c)=\infty$ and $\lim_{c\to\infty} v'(c)=0$.

(B.2) $B \in C^2(\mathbb{R}_{++}\times[0,1],\mathbb{R}_{++})$ with $B(Y,s)>1$ and $B_Y(Y,s)>0>B_s(Y,s)$.

(B.3) $\dfrac{B(Y,s)(2B_Y(Y,s)y+B_{YY}(Y,s)y^2)}{(B(Y,s)+B_Y(Y,s)y)(B(Y,s)+B_Y(Y,s)y)}, \dfrac{B(Y,s)(B_s(Y,s)+B_{Ys}(Y,s)y)}{(B(Y,s)+B_Y(Y,s)y)B_s(Y,s)} \leq 1$ for $Y=y$.

Assumption (B.1) states that consumers are risk averse with relative risk aversion greater than one. It is satisfied for CRRA utility functions $v(c)=(c^{1-\xi}-1)/(1-\xi)$, with $\xi>1$. Assumption (B.2) states that output of bank j is increasing in aggregate investment Y and decreasing in the state s. Assumption (B.3) is a technical assumption, which together with (B.2) is satisfied for linear productions functions $B(Y,s)=1+\psi Y+\omega(1-s)$ as well as for translated Cobb-Douglas production functions $B(Y,s)=(Y+1)^{\psi}(2-s)^{\omega}$, both with $\psi,\omega>0$.

3.2 Equilibrium

As standard in this class of banking models, consumers face an idiosyncratic risk at date $T=0$ as they do not know when they need to consume, and banks facilitate an efficient sharing of those risks. They do so by creating liquidity, which is the transformation of long-term productive investments into deposits that allow consumers to withdraw before maturity of those investments. Following conventions, banks are said to create more liquidity if the promised payout upon withdrawing at date $T = 1$ is larger.

It is useful to look at the feasibility constraints for any banking arrangement

$$
\begin{aligned}
tc_1 &\leq x, \\
(1-t)c_2(s) &\leq B(Y,s)y + x - tc_1 \ \text{ for all } s,
\end{aligned}
$$

They require payouts to impatient consumers tc_1 at date $T = 1$ to be smaller than or equal to the amount stored at date $T = 0$ and payouts to patient consumers $(1-t)c_2(s)$ at date $T = 2$ to be smaller than or equal to the amount produced at date $T = 2$ and the amount stored at date $T = 1$. Since production is more productive than storage in all states, the amount stored x at date $T = 0$ is fully used for payouts to impatient consumers. Consequently, investment y determines the consumption profile (c_1, c_2) and the constraints reduce to

$$
\begin{aligned}
c_1 &= \frac{1-y}{t} \\
c_2(s) &= \frac{B(Y,s)y}{1-t}.
\end{aligned}
$$

Consequently, the banking economy is a special case of the Bayesian game studied in Section 2.

Observation 1 *Assume* (t, v, B) *satisfies (B.1)–(B.3). Then* (A, S, u) *with* y *as strategic variable,* $A = [0,1]$ *as strategy set, and,*

$$
\begin{aligned}
u_i(y,Y,s) &= t\,v\left(\frac{1-y}{t}\right) + (1-t)\,v\left(\frac{B(Y,s)y}{1-t}\right) \\
u(Y,s) &= t\,v\left(\frac{1-Y}{t}\right) + (1-t)\,v\left(\frac{B(Y,s)Y}{1-t}\right)
\end{aligned}
$$

as payoff functions for banks and the central bank, respectively, satisfies (A.1)–(A.5).

Proof: See Appendix A. □

The observation implies all results produced in the previous section directly apply to the banking economy. Let $y^{\sigma_{\text{FR}}}(s)$, respectively $y^{\sigma_{\text{NR}}}(0)$, be the unique symmetric Nash equilibrium under a fully revealing communication strategy, respectively the unique symmetric Nash equilibrium under the non-revealing communication strategy, for signal s. Then the equilibrium strategies satisfy

$$\begin{aligned} -v'\left(\frac{1-y^{\sigma_{\text{FR}}}(s)}{t}\right)+B(y^{\sigma_{\text{FR}}}(s),s)v'\left(\frac{B(y^{\sigma_{\text{FR}}}(s),s)y^{\sigma_{\text{FR}}}(s)}{1-t}\right) &= 0 \\ -v'\left(\frac{1-y^{\sigma_{\text{NR}}}(0)}{t}\right)+\int_0^1 B(y^{\sigma_{\text{NR}}}(0),s)v'\left(\frac{B(y^{\sigma_{\text{NR}}}(0),s)y^{\sigma_{\text{NR}}}(0)}{1-t}\right)\mathrm{d}s &= 0 \end{aligned} \tag{3}$$

and according to Theorem 2 a two-steps partional equilibrium pinned down by a threshold state $s_1 \in (0,1)$ exists provided

$$u(y^{\sigma_{\text{FR}}}(0),y^{\sigma_{\text{FR}}}(0),0) \;>\; u(y^{\sigma_{\text{NR}}}(0),y^{\sigma_{\text{NR}}}(0),0).$$

Similarly, let $y^P(s)$ be the investment level the central bank finds optimal in state s, solving

$$-v'\left(\frac{1-y^P(s)}{t}\right)+\left(B'_Y(y^P(s),s)y^P+B(y^P(s),s)\right)v'\left(\frac{B(y^P(s),s)y^P(s)}{1-t}\right) \;=\; 0. \tag{4}$$

According to Lemma 4, $y^P(s) > y^{\sigma_{FR}}(s)$. The reason is the central bank takes the investment externality into account, but banks do not.

It can be shown that for CRRA utility functions and linear production functions, the parameters relating to preferences and technologies determine whether equilibria featuring a partitional two-steps communication exist.

Observation 2 *Suppose* $v(c)=c^{1-\xi}/(1-\xi)$ *with* $\xi>1$*, and* $B(Y,s)=1+\psi Y+\omega(1-s)$ *with* $\psi,\omega>0$*. Then, there is* $\bar{\xi}:\mathbb{R}^2_+\to\mathbb{R}_{++}\backslash]0,1[$*,* $\bar{\psi}:\mathbb{R}_+\times\mathbb{R}_{++}\backslash]0,1[\to\mathbb{R}_{++}$*, and* $\bar{\omega}:\mathbb{R}_{++}\backslash]0,1[\times\mathbb{R}_+\to\mathbb{R}_{++}$*, such that that a two-steps partional equilibrium exists if* $\xi>\bar{\xi}(\psi,\omega)$ *or* $\psi<\bar{\psi}(\xi,\omega)$ *or* $\omega>\bar{\omega}(\xi,\psi)$*.*

The observation follows from inspecting the two first-order conditions (3) and (4) using the functional forms. An equilibrium exists if the risk aversion is high, the externality is weak and uncertainty about productivity is high. If the risk aversion is high, consumers are less willing to bear the risk associated with production when patient. Similarly, if the state of the economy matters a lot, then the risk associated with production is important. In equilibrium, a two-steps communication strategy reduces the consumers' risk exposure in both cases. Finally, if the externality is weak, then inefficiencies from revealing the true state are small and in an equilibrium with a two-steps communication strategy, the central bank indeed provides more detailed information than in a non-revealing communication strategy.

An equilibrium message s_1 implies $s \leq s_1$ (i.e. high productivity) while equilibrium message 1 implies $s > s_1$ (i.e. low productivity). If $y^{\sigma}(s_1)$ and $y^{\sigma}(1)$ are the investment levels under these two messages of high and low productivity, then

$$\begin{aligned} -v'\left(\frac{1-y^{\sigma}(s_1)}{t}\right)+\frac{1}{s_1}\int_0^{s_1} B(y^{\sigma}(s_1),s)v'\left(\frac{B(y^{\sigma}(s_1),s)y^{\sigma}(s_1)}{1-t}\right)\mathrm{d}s &= 0 \\ -v'\left(\frac{1-y^{\sigma}(1)}{t}\right)+\frac{1}{1-s_1}\int_{s_1}^{1} B(y^{\sigma}(1),s)v'\left(\frac{B(y^{\sigma}(1),s)y^{\sigma}(1)}{1-t}\right)\mathrm{d}s &= 0. \end{aligned} \tag{5}$$

According to Lemma 3, $y^{\sigma}(s_1) < y^{\sigma}(1)$. Obviously, patient consumers directly benefit from higher productivity for a fixed investment. However, since the risk aversion is larger than one, consumers want to benefit from higher productivity also in case they become impatient. Hence, in an equilibrium featuring a two-steps communication strategy, banks create more liquidity if the prospects of the economy are pronounced good than if the prospects of the economy are pronounced bad, whereby the former is associated with more storage and less investment than the latter. In particular, if the state is in the neighborhood of the threshold state s_1 and $\mathscr{C}$ pronounces good prospects of the economy, then banks create too much liquidity compared to what $\mathscr{C}$ would like them to create.

In sum, Theorem 1 implies central banks cannot perfectly reveal its information, while Theorem 2 implies partially revealing communication is possible in equilibrium. The combination of the two results thus explains why central banks are vague in their communication.

3.3 Information and welfare

The equilibrium communication strategy typically does not maximise the expected welfare of the economy. An equilibrium featuring partitional two-steps communication exists provided there is $s_1 \in]0,1[$ satisfying:

$$u(y^{\sigma}(s_1), y^{\sigma}(s_1), s_1) = u(y^{\sigma}(1), y^{\sigma}(1), s_1). \tag{6}$$

The expected welfare is at its maximum provided the threshold state s_1 satisfies

$$\begin{aligned} s_1 \ \in \ \arg\max \quad & \left\{ \int_0^{s_1} tv\left(\frac{1-y^{\sigma}(s_1)}{t}\right) + (1-t)v\left(\frac{B(y^{\sigma}(s_1),s)y^{\sigma}(s_1)}{1-t}\right) \mathrm{d}s \right. \\ & \left. + \int_{s_1}^{1} tv\left(\frac{1-y^{\sigma}(1)}{t}\right) + (1-t)v\left(\frac{B(y^{\sigma}(1),s)y^{\sigma}(1)}{1-t}\right) \mathrm{d}s \,\middle|\, s_1 \in [0,1] \right\}, \end{aligned}$$

again with $y^{\sigma}(s_1)$ and $y^{\sigma}(1)$ satisfying Eq. (5). Let τ^* denote the threshold (defining a two steps partitional strategy) that maximises expected welfare. If $\tau^* \in]0,1[$, then τ^* describes the expected welfare maximising two-step partitional communication strategy provided it satisfies the first-order condition:

$$\begin{aligned} & \left(tv\left(\frac{1-y^{\sigma}(\tau^*)}{t}\right) + (1-t)v\left(\frac{B(y^{\sigma}(\tau^*),\tau^*)y^{\sigma}(\tau^*)}{1-t}\right)\right) \\ & \quad - \left(tv\left(\tfrac{1-y^{\sigma}(1)}{t}\right) + (1-t)v\left(\frac{B(y^{\sigma}(1),\tau^*)y^{\sigma}(1)}{1-t}\right)\right) \\ & \qquad + \int_0^{\tau^*} (1-t)v'\left(\frac{B(y^{\sigma}(\tau^*),s)y^{\sigma}(\tau^*)}{1-t}\right) \frac{B'_Y(y^{\sigma}(\tau^*),s)y^{\sigma}(\tau^*)}{1-t} \frac{\mathrm{d}y^{\sigma}(\tau^*)}{\mathrm{d}\tau^*} \mathrm{d}s \\ & \qquad\quad + \int_{\tau^*}^{1} (1-t)v'\left(\frac{B(y^{\sigma}(1),s)y^{\sigma}(1)}{1-t}\right) \frac{B'_Y(y^{\sigma}(1),s)y^{\sigma}(1)}{1-t} \frac{\mathrm{d}y^{\sigma}(1)}{\mathrm{d}\tau^*} \mathrm{d}s \ = \ 0. \end{aligned} \tag{7}$$

Therefore, if Eq. (6) holds, then the left side in Eq. (7) is strictly positive provided the threshold state $s_1 = \tau^*$ were the same. According to Theorem 3, the solutions to Eqs. (6) and (7) thus imply $\tau^* > s_1$ if a central bank with the preferences of a representative consumer is tasked with communicating.

If instead $\tau^* \in \{0,1\}$, then expected welfare is higher with a non-revealing communication strategy than with a two-steps communication strategy. A necessary condition for non-revealing communication strategies to be indeed welfare-optimal is, however, that they outperform a fully revealing communication strategy. It can be shown that for CRRA utility functions and linear production functions, the comparative advantage of a fully revealing communication strategy over a non-revealing strategy depends on parameters relating to preferences and technologies as follows.

Observation 3 *Suppose* $v(c) = c^{1-\xi}/(1-\xi)$ *with* $\xi > 1$*, and* $B(Y,s) = 1 + \psi Y + \omega(1-s)$ *with* $\psi, \omega > 0$*. Then, for* ξ *or* ω *sufficiently large or* ψ *sufficiently small, a fully revealing communication strategy outperforms a non-revealing communication strategy in terms of expected consumer welfare.*

This Observation follows from inspecting the respective first-order conditions (3) and (4) using the functional forms, which imply that $y^{\sigma_{\text{FR}}}(s) - y^P(s)$ converges to zero for $\xi \to \infty$, $\omega \to \infty$ or $\psi \to 0$. The key is that in the limit, the policy that is implemented by banks in a fully revealing equilibrium converges to the planner's optimal policy. It follows immediately that in the limit, the planner consequently prefers full revelation over any partially revealing or non-revealing communication.

3.4 Delegated communication

Since Corollary 2 applies here, consumers' expected welfare can be increased by delegating communication to a central bank that finds productive investments less desirable. In the present context, this translates into treating impatient consumers more favorably than patient consumers. When consumers maximise their expected utility, they put a weight on their utility as impatient consumer that is equal to the probability t of becoming impatient. Conversely, their weight on their utility as patient consumer is equal to the probability $1-t$ of turning out patient. Suppose communication is delegated to a central bank that does not share the interests of consumers but places a relative weight $b \in]0,1[$ on impatient consumers.

Observation 4 *Assume* (t, v, B) *satisfies* (B.1)*,* (B.2) *and* (B.3)*. Suppose there is a two-steps partitional equilibrium pinned down by* $\bar{s}_1 \in]0,1[$*. Then consumers' expected welfare can be increased by delegating communication to a central bank putting more weight on impatient consumers* $b \in]t,1[$*.*

Proof: State $\bar{s}_1$ is a threshold for a two-step partitional equilibrium provided

$$bv\left(\frac{1-y^{\sigma}(\bar{s}_1)}{t}\right)+(1-b)v\left(\frac{B(y^{\sigma}(\bar{s}_1),\bar{s}_1)y^{\sigma}(\bar{s}_1)}{1-t}\right) = bv\left(\frac{1-y^{\sigma}(1)}{t}\right)+(1-b)v\left(\frac{B(y^{\sigma}(1),\bar{s}_1)y^{\sigma}(1)}{1-t}\right).$$

Moreover, $v((1-y^{\sigma}(\bar{s}_1))/t) > v((1-y^{\sigma}(1))/t)$ and $v(B(y^{\sigma}(\bar{s}_1),\bar{s}_1)y^{\sigma}(\bar{s}_1)/(1-t)) < v((B(y^{\sigma}(1),\bar{s}_1)y^{\sigma}(1))/(1-t))$ because $y^{\sigma}(\bar{s}_1) < y^{\sigma}(1)$. Hence, if the consumers' expected welfare is maximised, then

$$tv\left(\frac{1-y^{\sigma}(\bar{s}_1)}{t}\right)+(1-t)v\left(\frac{B(y^{\sigma}(\bar{s}_1),\bar{s}_1)y^{\sigma}(\bar{s}_1)}{1-t}\right) < tv\left(\frac{1-y^{\sigma}(1)}{t}\right)+(1-t)v\left(\frac{B(y^{\sigma}(1),\bar{s}_1)y^{\sigma}(1)}{1-t}\right)$$

holds because $u(y^{\sigma}(\bar{s}_1), y^{\sigma}(\bar{s}_1), \bar{s}_1) < u(y^{\sigma}(1), y^{\sigma}(1), \bar{s}_1)$. Therefore, there is $b \in]t,1[$ such that above condition holds. □

Recall that $y^{\sigma_{FR}}(s) < y^P(s)$ for all s due to the externality so banks create too much liquidity compared to what $\mathscr{C}$ would like them to create for the fully revealing signal. Yet, expected welfare of consumers can be improved by delegating communication about the state to a central bank that prefers liquidity creation even more than consumers do themselves. Thereby the objective of the central bank becomes on the one hand less aligned with the aim of $\mathscr{C}$ and on the other hand more aligned with the objectives of the individual consumers. Consequently, central bank communication becomes more credible and on average its signal improves liquidity creation from a welfare perspective.

The following example further illustrates our results from this application. Suppose $v(c) = (c^{1-\xi} - 1)/(1-\xi)$ and $B(Y,s) = 1 + \psi Y + \omega(1-s)$ with $t = 0.5$, $\xi = 2$, $\psi = 0.1$, and $\omega = 9$. Then $y^{\sigma_{\mathrm{FR}}}(0) \approx 0.24003$, $y^P(0) \approx 0.24025$, $y^{\sigma_{\mathrm{NR}}}(0) \approx 0.33446$, and $u(y^{\sigma_{\mathrm{NR}}}(0), y^{\sigma_{\mathrm{NR}}}(0), 0) - u(y^{\sigma_{\mathrm{FR}}}(0), y^{\sigma_{\mathrm{FR}}}(0), 0) \approx 0.01727 > 0$. The condition in Theorem 2 is thus satisfied, and a partially revealing two-step equilibrium exists, characterized by the partitional strategy σ such that $s_1 \approx 0.7077$, $y^{\sigma}(s_1) \approx 0.2846$, and $y^{\sigma}(1) \approx 0.4091$. Expected welfare would be highest if the threshold were approximately equal to 0.719. Such a threshold value implies $y^{\sigma}(s_1) \approx 0.2858$ and $y^{\sigma}(1) \approx 0.4111$. This threshold characterises a two-steps partitioning equilibrium if communication is delegated to someone whose weight on impatient consumers is about 1.0071 times higher than the weight consumers themselves put on becoming impatient. Finally, note that welfare is not maximised at $s_1 = 0.5$ where the Shannon entropy to measure the informational value of a two-step signal is at its maximum. Indeed, the welfare-maximising two-steps communication strategy implies an even smaller entropy (0.5939) than that achieved without delegation (0.6042).

4 Communicating with Cournot oligopolists

In this section we consider cheap-talk in the Cournot model with demand uncertainty. We assume linear demand as well as identical and constant marginal costs. The simplicity of the model allows us to go beyond two-steps partional equilibria. We find that consumer and firm surplus depend exclusively on the informativeness of equilibrium communication. We identify the weighting of consumer and producer surplus that ensures existence of fully revealing equilibrium, thereby maximising the sum of expected consumer and producer surplus.

4.1 Setup and definitions

Consider a market with n firms competing à la Cournot by simultaneously setting quantities and facing an inverse demand $p(q) = s - bq$, where q is the total quantity supplied by firms. All firms have the same constant marginal cost c.[1] There is uncertainty among firms about parameter s, which is randomly drawn from a uniform on $[\underline{s}, \bar{s}]$, where $\underline{s} > c$. While firms only know the ex ante distribution of s, they face a sender $\mathscr{C}$ who observes the realized value of s. Before the quantity setting stage, $\mathscr{C}$ communicates publicly by sending a cheap talk message $m \in M = S$. $\mathscr{C}$'s utility, given s and a profile of quantities chosen by firms, is given by a weighted sum of producer surplus and consumer surplus, where weight γ is attached to producer surplus and weight $1 - \gamma$ is attached to consumer surplus. Thus, $\mathscr{C}$ aims to maximise total welfare in case $\gamma = 1/2$.

For a fixed natural number N, we denote by $\widetilde{\sigma}_{\mathrm{N}}$ the N-intervals partitional strategy featuring $s_i = \frac{i}{N}$. Denote by $|\sigma|$ the number of messages sent with positive probability according to σ. Denote by $E_\sigma[s|m]$ the conditional expected value of s, given that $\mathscr{C}$ has sent m and is known to use σ. Denote by $Var_\sigma(E_\sigma[s|m])$ the variance of the distribution of $E_\sigma[s|m]$ induced by σ.

Denote by $w_F(s^*, q)$, respectively $w_C(s^*, q)$, the total firm, respectively consumer, surplus given realised state $s = s^*$ and a total output of q, produced by each firm choosing

[1] In the Appendix, we consider the case where firms' cost function is convex and given by $\frac{c}{2}q_i^2$ and find that all our main results survive qualitatively.

output level q/n. Given s^* and q, the payoff of $\mathscr{C}$ is:

$$u_\gamma(q,s^*) = \gamma w_F(q,s^*) + (1-\gamma) w_C(q,s^*).$$

Denote by $q_{NE}(\widetilde{s})$ the unique symmetric Nash equilibrium aggregate output level given any conditional expectation $\widetilde{s}$ induced by $\mathscr{C}$'s message. Let $\hat{w}_F(\widetilde{s},s^*) = w_F(q_{NE}(\widetilde{s}),s^*)$ and $\hat{w}_C(\widetilde{s},s^*) = w_C(q_{NE}(\widetilde{s}),s^*)$. In a putative equilibrium, for realised state s^* and expectation $\widetilde{s}$ the payoff of $\mathscr{C}$ is:

$$\hat{u}_\gamma(\widetilde{s},s^*) = \gamma \hat{w}_F(\widetilde{s},s^*) + (1-\gamma)\hat{w}_C(\widetilde{s},s^*).$$

In an equilibrium featuring a partitional communication strategy σ, the ex ante expected payoff of $\mathscr{C}$ is:

$$\begin{aligned} U_\gamma(\sigma) &= \frac{1}{\bar{s}-\underline{s}} \int_{\underline{s}}^{\bar{s}} \hat{u}_\gamma(E[s\,|m_\sigma(s)],s)ds \\ &= \frac{1}{\bar{s}-\underline{s}} \int_{\underline{s}}^{\bar{s}} [\gamma \hat{w}_F(E[s\,|m_\sigma(s)],s) + (1-\gamma)\hat{w}_C(E[s\,|m_\sigma(s)],s)]\,ds. \end{aligned}$$

Using our notation, $U_1(\sigma)$ corresponds to ex ante expected firm profits and $U_0(\sigma)$ corresponds to ex ante expected consumer surplus.

4.2 Equilibrium

In the unique Nash equilibrium of the quantity setting subgame, each firm i sets quantity $q_{NE,i}(\widetilde{s}) = \frac{\widetilde{s}-nc_i+C_{-i}}{b(n+1)}$ given $E[s\,|m] = \widetilde{s}$. Total firm surplus given s^* and $\widetilde{s}$ equals:

$$\hat{w}_F(s^*,\widetilde{s}) = n\left(s^* - b\left(\frac{n(\widetilde{s}-c)}{b(n+1)}\right) - c\right)\left(\frac{\widetilde{s}-c}{b(n+1)}\right)$$

while consumer surplus is:

$$\hat{w}_C(s^*,\widetilde{s}) = \frac{b}{2}\left(\frac{(\widetilde{s}-c)\,n}{b(n+1)}\right)^2.$$

We now address what partitional communication strategies can be part of an equilibrium. The partitional strategy σ, pinned down by thresholds $s_0 = \underline{s} < s_1 < s_2 < .. < s_{N-1} < s_N = \overline{s}$ is part of a Perfect Bayesian equilibrium if and only for all $i, j \in \{1, \ldots, N\}$ with $j \neq i$, and all $s \in (s_{i-1}, s_i]$ (including s_{i-1} for $i = 1$), we have:

$$\hat{u}_\gamma(E_\sigma[s\,|s_i], s) \;\geq\; \hat{u}_\gamma(E_\sigma[s\,|s_j], s)$$

In words, $\mathscr{C}$ should never have a strict incentive to deviate from his equilibrium message. By standard arguments, it follows that a partitional strategy $\sigma = \{s_r\}_{r=0}^N$ is part of a Perfect Bayesian equilibrium if and only if it holds true that for all $i, j \in \{1, \ldots, N\}$ with $i \neq j$

$$\hat{u}_\gamma(E_\gamma[s\,|m = s_i], s_i) = \hat{u}_\gamma(E_\gamma[s\,|m \;=\; s_{i+1}], s_i).$$

In terms of achievable equilibrium communication, the key underlying aspect is $\mathscr{C}$'s incentive to bias firms' beliefs away from s for any given realisation of s. It can be shown that $\hat{u}_\gamma(s^*, \widetilde{s})$ is concave in the expectation $\widetilde{s}$ for any realized s. Given s, the $\mathscr{C}$-optimal $\widetilde{s}$ solves $\partial \hat{u}_\gamma(s, \widetilde{s})/\partial \widetilde{s} = 0$ and is given by

$$\widetilde{s}^*(s) = s + \frac{n + \gamma(1-2n)}{n\,(3\gamma - 1)}\,(s - c)\,.$$

The function is continuously increasing in γ. If $\mathscr{C}$ only cares about firm profits ($\gamma = 1$), then $\widetilde{s}^*(s) < s$ where $s - \widetilde{s}^*(s) = (s{-}c)(n{-}1)/(2n)$. The intuition is that firms in equilibrium overproduce compared to the total profit maximising quantity. Biasing downwards firms' beliefs allows to induce the total profits maximising monopoly output. If $\mathscr{C}$ cares only about consumer surplus, then instead $\widetilde{s}^*(s) = \overline{s}$. From a consumer perspective, a larger aggregate output is always better no matter s. If $\mathscr{C}$ cares equally about firms and consumers ($\gamma = \frac{1}{2}$), then $\widetilde{s}^*(s) > s$ where $\widetilde{s}^*(s) - s = (s{-}c)/n$. So an unbiased $\mathscr{C}$ has an incentive to overreport. The equality $\widetilde{s}^*(s) = s$ holds if and only if $\gamma = n/(2n{-}1)$. The intuition is simple: With

$n \geq 2$ firms, in a putative equilibrium with truth-telling, firms act in a way that, rather than maximise total profits, instead maximises the utility of a virtual planner whose payoff function is $\hat{u}_{\frac{n}{2n-1}}(s^*, \widetilde{s})$. Therefore, we can think of the communication game as $\mathscr{C}$ facing a virtual monopolist endowed with the objective function $\hat{u}_{n/(2n-1)}(s^*, \widetilde{s})$. Clearly, if (and only if) $\mathscr{C}$'s objective function is also $\hat{u}_{n/(2n-1)}(s^*, \widetilde{s})$, then $\mathscr{C}$ and the virtual monopolist share the same optimal decision rule, in which case $\mathscr{C}$ has no incentive to deviate from truth-telling in a putative truth-telling equilibrium. It follows immediately from the above that a truth-telling equilibrium exists if and only if $\gamma = n/(2n-1)$.

Observation 5

- *With $n \geq 2$ there is no fully revealing equilibrium for $\gamma = 1/2$.*
- *There is a fully revealing equilibrium if and only if $\gamma = n/(2n-1)$.*
- *For every $N \geq 2$ there is an equilibrium featuring $\widetilde{\sigma}_N$ if and only if $\gamma = n/(2n-1)$.*

We add some comments on the second point above. First, $\gamma = n/(2n-1)$ implies existence of a wide range of partitional equilibria in addition to the fully revealing equilibrium. For any s^* and $\delta > 0$, it can be shown that $\gamma = n/(2n-1)$ is the unique solution to the equality $\hat{u}_\gamma(s^*, s^* - \delta) = \hat{u}_\gamma(s^*, s^* + \delta)$. If (and only if) $\gamma = n/(2n-1)$, $\mathscr{C}$'s objective function thus exhibits a particular form of symmetry: For any given realised state s^*, $\mathscr{C}$ is indifferent between inducing any two conditional expectations of s that are equidistant from s^*. In terms of equilibrium, this trivially implies that all simple partitions $\widetilde{\sigma}_{\mathrm{N}}$ for $N \geq 2$ are equilibrium partitions if (and only if) $\gamma = n/(2n-1)$.

Instead of solving for the γ that ensures that a particular type of strategy (truth-telling or partitions with equally sized intervals) is part of an equilibrium, an alternative approach is to fix γ exogenously and solve for the implied two intervals equilibria, as we do next. The following is a Corollary of Theorem 2.

Observation 6 *For a fixed γ, there is a two steps signaling equilibrium if*

$$\hat{u}_\gamma(0,0) > \hat{u}_\gamma\left(0, \frac{\underline{s}+\overline{s}}{2}\right).$$

To pin down the threshold characterising the (unique) two intervals partitional equilibrium, we simply need to solve

$$\hat{u}_\gamma\left(s_1, \frac{\underline{s}+s_1}{2}\right) = \hat{u}_\gamma\left(s_1, \frac{s_1+\overline{s}}{2}\right),$$

for s_1. The above has a unique solution

$$s_1^* = \frac{4c\gamma - n(\underline{s}+\overline{s}) + 4cn - 8cn\gamma + 3n\gamma(\underline{s}+\overline{s})}{2n+4\gamma-2n\gamma} \tag{8}$$

Note furthermore that

$$\frac{ds_1^*}{d\gamma} = \frac{n(n+1)(\underline{s}-2c+\overline{s})}{(n+2\gamma-n\gamma)^2}$$

which is trivially positive for any $\gamma \in (0,1)$.

Setting $\gamma = 1/2$, the unique solution to (8) reduces to $s_1^* = (2c+(n/2)(\underline{s}+\overline{s}))/(n+2)$. Clearly, for $\gamma = 1/2$ a two intervals equilibrium exists if and only if $\underline{s} < s_1^* < \overline{s}$. Recalling our assumption $c < \underline{s}$, we can conclude that a two-intervals equilibrium exists if and only if $\underline{s}-(n/4)(\overline{s}-\underline{s}) < c < \underline{s}$. So an excessively low marginal cost makes such an equilibrium impossible. The intuition is that the smaller c, the more firms underproduce in $\mathscr{C}$'s eyes, and consequently the more $\mathscr{C}$ wants to bias firms's beliefs upwards. Recall indeed that as shown earlier $\widetilde{s}^*(s) - s = (s-c)/n$ for $\gamma = 1/2$. Note also that s_1^* is strictly smaller than $(\underline{s}+\overline{s})/2$ so the higher the realized state, the more precise is equilibrium communication in the sense that in equilibrium firms' ex post uncertainty is larger for very high state realisations than for very low state realisations.

We add some remarks on equilibria with more than two steps. Generally, given $\gamma = 1/2$ condition (4.2) implies the difference equation

$$s_{i+1} = \left(\frac{4+2n}{n}\right) s_i - s_{i-1} - \frac{4}{n} c, \tag{9}$$

with conditions $s_0 = \underline{s}$ and $s_N = \bar{s}$. By standard arguments, there exists a finite $\overline{N}$ such that there exists a unique equilibrium featuring an N-intervals partition for every $N \leq \overline{N}$. In the limit as n tends to infinity, (9) tends to $s_{i+1} = 2s_i - s_{i-1}$ which for every $N \geq 2$ admits the solution $s_i = \underline{a} + (i/N)(\bar{s} - \underline{s})$. Hence, for every $N \geq 2$, for n large enough there exists an equilibrum featuring a partition which is arbitrarily close to $\widetilde{\sigma}_N$. Fully revealing communication (i.e. $N = \infty$ is thus feasible in the limit. Indeed, as n increases, in a fully revealing equilibrium the aggregate output produced by firms for every s converges to the welfare maximising output level, in line with $\mathscr{C}$'s preferences.

4.3 Information and welfare

Besides clarifying what partitional communication strategies can be part of an equilibrium, a key question is the ex ante expected firm and consumer surplus, in a putative equilibrium featuring a given partitional communication strategy σ. We have:

Observation 7

- *For all partitional signaling strategies σ:*

$$U_0(\sigma) \; = \; \frac{n^2}{2b(n+1)^2}\left[Var_\sigma(E[s|m]) + (E[s]-c)^2\right] \; = \; \frac{n}{2}U_1(\sigma).$$

- *For all $\gamma \in (0,1)$ and every $N \geq 2$, $\widetilde{\sigma}_N = \underset{|\sigma| \leq N}{\arg\max} U_\gamma(\sigma)$ and $U_\gamma(\widetilde{\sigma}_{N+1}) > U_\gamma(\widetilde{\sigma}_N)$.*

Note that the chosen communication strategy σ solely affects $Var_\sigma(E[s|m])$ in both expressions $U_0(\sigma)$ and $U_1(\sigma)$. In order to maximise either $U_0(\sigma)$ or $U_1(\sigma)$, the communication strategy should maximise the informativeness in the sense of maximising the variance of the conditional expectation $E[s|m]$. We may thus conclude that in this model, for any γ and any partially informative partitional strategy σ it holds true that:

$$U_\gamma(\sigma_{\text{NR}}) < U_\gamma(\sigma) < U_\gamma(\sigma_{\text{FR}})$$

In other words, both firms and consumers prefer a partially revealing partitional strategy over no information transmission, and prefer perfect information transmission over partially revealing communication. It follows immediately that $\mathscr{C}$ has the same ranking of strategies whatever his bias γ.

Another implication is that if at most N messages can be used, then the optimal partitional strategy is the identical intervals partition $\widetilde{\sigma}_{\text{N}}$. Similarly, note that $Var_{\widetilde{\sigma}_{\text{N+1}}}(E[s|m]) > Var_{\widetilde{\sigma}_{\text{N}}}(E[s|m])$ for any $N \geq 1$. So conditional on using a partition with identically sized intervals, more intervals are always beneficial.

4.4 Delegated communication

When considered together, our previous observations have clear implications in terms of the optimal delegation choice, if $\mathscr{C}$ is interested in maximising total welfare (i.e. $\gamma = 1/2$).

Observation 8

- *Restricting attention to two intervals equilibria, $\mathscr{C}$ gains by delegating communication to an agent with bias $\gamma \in]1/2, (1/2)(n^2+3n)/(n^2+2n-2)[$ where $(1/2)(n^2+3n)/(n^2+2n-2) > n/(2n-1)$.*

- *Assume that at most N messages can be used in equilibrium. $\mathscr{C}$'s expected payoff is maximised if $\mathscr{C}$ delegates communication to an agent with with bias $\gamma = n/(2n-1)$, and the ensuing equilibrium played features the partitional communication strategy $\widetilde{\sigma}_N$.*

Concerning the first point, $\mathscr{C}$ gains by delegating communication to an agent with a bias γ located in the designated interval because the implied two-intervals equilibrium features a threshold s_1 which is closer to the optimal value $\frac{\underline{s}+\bar{s}}{2}$ than if he does not delegate. As to the second point, the idea is that for any $N \geq 2$, an agent with bias $\gamma = n/(2n-1)$ can achieve the welfare optimal N-intervals partitional equilibrium. In the absence of any restriction on N, this agent can achieve fully revealing communication ($N = \infty$), which is the best possible outcome.

5 Final remarks

We have studied cheap-talk models with a single sender and multiple market players. The sender aims at maximising expected aggregate welfare of the market players and possibly some other group of agents influenced by the actions of the market players. However, there are externalities in that the action of every market player influences the welfare of other market players. These externalities render perfect information transmission impossible even if the sender aims to maximise aggregate received welfare.

We have considered two applications, namely a banking model with production externalities and a Cournot model. For the banking model, externalities provide an explanation for why central banks are vague in their announcements and communication is better delegated to an agent who wants banks to create more liquidity. For the Cournot model, pecuniary externalities between firms provide a rationale for delegating communication to an agent who is more concerned about firms than consumers, though the planner cares equally about both parties.

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Appendix A: Proof of Observation 1

- Assumption (A.1):

$$\begin{aligned}
\frac{\partial u_i(y,Y,s)}{\partial y_i} &= -v'\left(\frac{1-y_i}{t}\right)+B(Y,s)v'\left(\frac{B(Y,s)y_i}{(1-t)}\right)\\
\lim_{y\to 0}\left.\frac{\partial u_i(y,Y,s)}{\partial y}\right|_{y=Y} &= -v'\left(\frac{1}{t}\right)+B(0,s)v'\left(\frac{0}{(1-t)}\right)>0\\
\lim_{y\to 1}\left.\frac{\partial u_i(y,Y,s)}{\partial y}\right|_{y=Y} &= -v'\left(\frac{0}{t}\right)+B(1,s)v'\left(\frac{B(1,s)}{(1-t)}\right)<0\\
\left.\frac{\partial^2 u_i(y,Y,s)}{\partial y\partial y}\right|_{y=Y} &= \frac{1}{t}v''\left(\frac{1-Y}{t}\right)+\frac{(B(Y,s))^2}{(1-t)}v''\left(\frac{B(Y,s)Y}{(1-t)}\right)<0
\end{aligned}$$

holds according to (B.1) and (B.2).

- Assumption (A.2)

$$\begin{aligned}
\left.\frac{\partial^2 u_i(y,Y,s)}{\partial y\partial Y}\right|_{y=Y} &= v'\left(\frac{B(Y,s)Y}{(1-t)}\right)B'_Y(Y,s)\left(1-R\left(\frac{B(Y,s)Y}{(1-t)}\right)\right)<0\\
\left.\frac{\partial^2 u_i(y,Y,s)}{\partial y\partial s}\right|_{y=Y} &= v'\left(\frac{B(Y,s)Y}{(1-t)}\right)B'_s(Y,s)\left(1-R\left(\frac{B(Y,s)Y}{(1-t)}\right)\right)>0
\end{aligned}$$

holds since $R\left(\frac{B(Y,s)Y}{1-t}\right)>1$ according to (B.1);

- Assumption (A.3)

$$\left.\frac{\partial u(y,Y,s)}{\partial y}\right|_{y=Y} = -v'\left(\frac{1-Y}{t}\right) + \left(B(Y,s) + B'_Y(Y,s)Y\right)v'\left(\frac{B(Y,s)Y}{(1-t)}\right)$$
$$> \left.\frac{\partial u_i(y,Y,s)}{\partial y_i}\right|_{y=Y}$$

holds since $B'_Y(Y,s) > 0$ according to (B.2)

- Assumption (A.4)

$$\lim_{y\to 0}\left.\frac{\partial u(y,Y,s)}{\partial y}\right|_{y=Y} = -v'\left(\frac{1}{t}\right) + B(0,s)v'\left(\frac{0}{(1-t)}\right) > 0$$
$$\lim_{y\to 1}\left.\frac{\partial u(y,Y,s)}{\partial y}\right|_{y=Y} = -v'\left(\frac{0}{t}\right) + \left(B(1,s) + B'_Y(1,s)\right)v'\left(\frac{B(1,s)}{(1-t)}\right) < 0$$

holds according to (B.1) and (B.2).

- Assumption (A.5)

$$\left.\frac{\partial^2 u(y,Y,s)}{\partial y \partial s}\right|_{y=Y} = v'\left(\frac{B(Y,s)Y}{(1-t)}\right)\left(B'_s(Y,s) + B''_{Ys}(Y,s)Y - \frac{B(Y,s)+B'_Y(Y,s)Y}{B(Y,s)/B'_s(Y,s)}R\left(\frac{B(Y,s)Y}{(1-t)}\right)\right)$$
$$< 0$$

The first part of (A.5) is satisfied because $\partial^2 u(y,y,s)/\partial y \partial y < 0$ provided the relative risk aversion is larger than one and the first part of (B.3) is satisfied. The second part of (A.5) is satisfied because $\partial^2 u(y,y,s)/\partial y \partial s$ is negative provided the relative risk aversion is larger than one and the second part of (B.3) is satisfied.

Appendix B: Proofs for Section 4

B.1 Linear cost function

B.1.1 Quantity setting subgame

Denote by q the total quantity produced while $q_{-i} = q - q_i$ is the sum of quantities produced by other firms. Let $C = nc$ while $C_{-i} = (n-1)c$. The inverse demand function writes $p(q_i, q_{-i}) = a - bq_{-i} - bq_i$. Given available information, firm i chooses quantity q_i to maximise its expected profit function

$$E_i\pi_i = E_i[(a - b(q_{-i} + q_i))\, q_i - c_i q_i]$$

The FOC for profit maximisation of i yields

$$q_i = \frac{1}{2b}(E_i[a] - c_i - bq_{-i}).$$

Summing the optimality equations derived for all n firms, we obtain:

$$\sum_{i=1}^{n} q_i = \frac{1}{2b}\left(nE_i[a] - \sum_{i=1}^{n} c_i - b\sum_{i=1}^{n} q_{-i}\right).$$

As we assume that information is public and all agents have the same priors, so we replace $E_i[a]$ by $E[a]$ next. Hence, the above rewrites as:

$$q^* = \frac{1}{2b}(nE[a] - C - b(n-1)q^*).$$

Solving for the above for q, we obtain the unique equilibrium solution $q^* = \frac{nE[a]-C}{b(n+1)}$. Solving for individual firms' output, we obtain $q_i^* = \frac{E[a]-c}{b(n+1)}$. We now want to compute firm i's profit in equilibrium given realised value a and expectation $E[a]$. Recalling that the equilibrium price p^* equals $a - bq^*$, firm i's profit given a and $E[a]$ is:

$$\pi_i^* = \left[a - b\left(\frac{nE[a]-nc}{b(n+1)}\right) - c\right]\left(\frac{E[a]-c}{b(n+1)}\right).$$

Consumer welfare is given by:

$$\begin{aligned}\int_0^{q^*}(a-bx)dx - p^*q^* &= \frac{1}{2}[a-p^*]\,q^* \\ &= \frac{b}{2}\left(\frac{(E[a]-c)\,n}{b(n+1)}\right)^2\end{aligned}$$

B.1.2 Ex ante expected utility

$$\begin{aligned}U_0(\sigma) &= \sum_{i=1}^{N} P(a \in (t_{i-1}, t_i])\left[\frac{b}{2}\left(\frac{(E[a|m_i]-c)\,n}{b(n+1)}\right)^2\right] \\ &= \frac{n^2}{2b(n+1)^2}\sum_{i=1}^{N} P(a \in (t_{i-1}, t_i])\left[\begin{array}{c}(E[a|m_i])^2 - 2E[a|m_i]E[a] + (E[a])^2 \\ +2E[a|m_i]E[a] - (E[a])^2 \\ -2E[a|m_i]c + c^2\end{array}\right] \\ &= \frac{n^2}{2b(n+1)^2}\sum_{i=1}^{N} P(a \in (t_{i-1}, t_i])\left[\begin{array}{c}(E[a|m_i]-E[a])^2 \\ +2E[a|m_i]E[a] - (E[a])^2 - 2E[a|m_i]c + c^2\end{array}\right] \\ &= \frac{n^2}{2b(n+1)^2}\left[Var(E[a|m]) + (E[a]-c)^2\right].\end{aligned}$$

The computation of $U_1(\sigma)$ follows similar steps.

B.2 Convex cost function

Assume here that firms' cost function is convex and given by $\frac{c}{2}q_i^2$. A standard FOC yields firm i's best response function:

$$q_i = \frac{(E_i[a] - bq_{-i})}{2b+c}.$$

Multiplying both sides by n, we obtain

$$nq = \frac{n\,(E_i[a] - bq_{-i})}{2b+c}.$$

Note that nq_{-i} equals $(n-1)q$, so that we have

$$q = \frac{nE[a]}{2b+c} - \frac{b(n-1)q}{2b+c}.$$

Solving the above for q yields

$$q^* = \frac{E[a]n}{b(1+n)+c}.$$

For any firms' production to be positive, we need $E[a] > bq_{-i}$, which is easily shown to always be true as $E[a] - bq^* = \frac{E[a](b+c)}{b+c+bn}$. Clearly, total profit given a and $E[a]$ is given by:

$$W_F(a, E[a]) = \left(a - \left(b + \frac{c}{2n}\right)\left(\frac{E[a]n}{b(1+n)+c}\right)\right)\left(\frac{E[a]}{b(1+n)+c}\right).$$

Consumer welfare is instead:

$$W_C(a, E[a]) = \int_0^{q^*} (a - bx)dx - p^* q^* = \frac{1}{2}\left[a - p^*\right] q^* = \frac{b}{2}\left(\frac{E[a]n}{b(1+n)+c}\right)^2.$$

It is easily shown that:

$$U_1(\sigma) = \frac{n\,(2b+c)}{8\,(b+c+bn)^2}\left[4Var(A) + (\underline{a} + \overline{a})^2\right] = \frac{(2b+c)}{bn} U_0.$$

It is easily checked that $u_\gamma(a,\widetilde{a})$ is concave in $\widetilde{a}$ for any a. Assuming $\gamma = \frac{1}{2}$, from $\mathscr{C}$'s perspective, for any a the optimal conditional expectation $\widetilde{a}$ is given by:

$$\widetilde{a}^*(a) = a + \frac{ab\,(n+\gamma-2n\gamma)}{c\gamma - bn + 3bn\gamma}.$$

It can be checked that $\widetilde{a}^*(a) \lesseqgtr a$ if $\gamma \gtreqless \frac{n}{2n-1}$. Solving $U_\gamma(a^*, a^*-\delta) = U_\gamma(a^*, a^*+\delta)$ gives the unique solution $\gamma = \frac{n}{2n-1}$. Solving for a two intervals partitional equilibrium $U_{\frac{1}{2}}\left(a^*, \frac{\underline{a}+t_1}{2}\right) = U_{\frac{1}{2}}\left(a^*, \frac{t_1+\overline{a}}{2}\right)$ gives the unique solution $t_1^* = \frac{(c+bn)(\overline{a}+\underline{a})}{4b+2(c+bn)}$.