

Governmental activity and private capital adjustment

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Abstract

We analyze within a dynamic model how firms decide on capital investment if the accompanying adjustment costs are a function of governmental activity. The government provides a public input and decides on the degree of rivalry. The productive public input enhances private capital productivity and reduces adjustment costs. We derive the equilibrium in which capital and investment ratio are both constant, carry out comparative dynamic analysis and discuss the model's policy implications. Increasing the amount of the public input unequivocally spurs capital investment whereas the result becomes ambiguous with respect to the impact of rivalry. Since a reduction in congestion increases the individually available amount of the public input, crowding out effects may lead to a reduction in the equilibrium capital stock. Most of the analysis is conducted for general production functions, although the case of CES technology is also considered.

JEL-codes: D21, H40, H54, O16.

Keywords: Governmental activity; congested public inputs; adjustment costs.

1 Introduction

The impact of governmental activity on firm investment has been extensively studied in the last several years. Beginning with the seminal work of Aschauer (1989a, 1989b) or Barro (1990) more recent models analyze the growth impact of governmental activity and include aspects of congestion, uncertainty, or excludability (see e. g. Fisher and Turnovsky (1998), Turnovsky (1999a, 1999b), Turnovsky (2000a) or Ott and Turnovsky (2006)). Within these models governmental activity consists of two parts: First, the provision of a productive input and second, the choice of the financing scheme that is required to provide a certain amount of the input as well as to internalize external effects of capital accumulation that arise for a given degree of congestion. But in addition to this governmental activity also includes non-fiscal instruments, e. g. the implementation of legislation and thus to define the firm's institutional environment (see e. g. Knack and Keefer (1995) or more Acemoglu et al. (2001, 2005)). Thus public policy plays an important role in the firm's capital investment decision via several channels.

While simple models assume that output might be transferred without additional costs into private capital the literature on investment theory which derives from the 'Tobin q' theory focuses on the impact of adjustment costs that arise e. g. due to an increase in demand. A survey of relevant approaches is given by Hamermesh and Pfann (1996) or Cooper and Haltiwanger (2003) whereas recent empirical studies can be found in Hall (2004). An industry specific discussion is done by Caballero and Engel (1999). Usually, those authors who focus on capital adjustment costs model them as relation between the investment in each period and the firm's capital stock. An exception is the paper of Turnovsky (1996) who develops a one-sector endogenous growth model in which capital investment incurs adjustment costs that are related to governmental activity. This picks up the argument that

firm specific aspects are not the unique determinants of capital adjustment. Aside from them also the economic environment like governmental activity gains importance.

Our paper merges both strands of the literature. While most models mentioned analyze the role of fiscal policy for capital accumulation we focus on the importance of the characteristics of the public input within the firm's investment process. We assume that private investment incurs adjustment costs that depend, among others, on the public input. While the amount provided is the outcome of a fiscal policy decision the prevailing degree of congestion may be interpreted as the outcome of an institutional policy decision (see Turnovsky (1996)). What we have in mind is the following: Governmental activities –here interpreted as being the provision of freely available infrastructure– have multiple impacts on investment in physical capital. First, the amount of the public input gains importance, e. g. capital productivity is higher in regions that are well endowed with infrastructure or the firms' overall investment costs are lower if the factory area is already developed. Then the public input not only enhances private capital productivity but also decreases adjustment costs that arise within the investment process. Second, the availability of the public input may be influenced by the government. This can be illustrated if e. g. the degree of congestion is reduced as consequence of driving bans for certain routes or the implementation of user fees that reduce private demand for infrastructure. To sum up, both –amount of infrastructure and degree of rivalry– are the consequence of governmental activity and become especially important to assess the impact of public policy on the firm's investment process.

The argumentation is incorporated within a dynamic model and illustrates that the hitherto existing focus on the absolute amount of publicly provided infrastructure and the analysis of the fiscal instruments oversimplifies the context. Since the degree of congestion determines the individually available amount of the public input it can be shown that rivalry

has an important and ambiguous impact on the capital investment decision.

We analyze within an dynamic model how a firm decides on capital investment if the accompanying adjustment costs are a function of the firm's investment and the governmental activity. Furthermore we include congestion effects into the model as introduced by Edwards (1990) or Glomm and Ravikumar (1994) and also incorporated by Turnovsky (1996). Hence the individually available amount diminishes with an increase in aggregate economic activity. The impact of the public input within the dynamics of the model is twofold: On the one hand it enhances productivity of private capital. On the other hand the adjustment costs are reduced by the extent of the available public input. With this respect our setup allows to disentangle the economic implications of infrastructure on the private investment decision in a production effect and an adjustment cost effect. Carrying out comparative dynamics we show that a better regional endowment with infrastructure unequivocally spurs capital investment via the production and the adjustment cost effect. An ambiguous impact results from rivalry: A reduction of congestion also reduces the adjustment costs and with this stimulates private capital accumulation. But at the same time the marginal productivity of governmental expenditures increases due to enhanced individual availability and this leads to a crowding out of capital investment. The incentive for capital accumulation diminishes and the impact of the production effect on equilibrium capital becomes negative.

The paper is organized as follows: After specifying the assumptions and the central economic effects within Section 2 the private investment decision is analyzed in Section 3. Then the first-order conditions are discussed. The equilibrium together with the transitional dynamics is derived in Section 5. Subsequently the implications of the public input on the resulting optimal capital stock and the investment ratio are presented in Section 6. Within Section 7 the policy implication of the model are discussed and the equilibrium

capital stock is calibrated for alternative parameter constellations in the context of a CES production function. We close with a short summary while the Appendix includes the formal derivations of some equations that are used within the paper.

2 The model

The individual firm i produces output, Y_i , using private capital, K_i , labour, L_i , and a governmental public input G_s . The production technology is linear-homogenous in the private inputs and

$$Y_i(t) = F(K_i(t), L_i(t), G_s(t)) = L_i(t) \cdot f(k(t), G_s(t)) \quad , \quad (1)$$

where f denotes output per capita and $k(t) \equiv \frac{K_i(t)}{L_i(t)}$.¹ The terms f_1 and f_2 denote the partial derivatives of f with respect to the inputs k and G_s . They are positive but decreasing ($f_1 > 0, f_2 > 0, f_{11} < 0, f_{22} < 0$) and the identical cross products $f_{12} = f_{21}$ are positive. Population is constant and consists of L individuals.² The firms are not charged for the use of the public production input but rivalry may arise. For example, various governments provide the road network without any fee, and the road network serves as a productive input for the firms in these countries. Nevertheless, there is congestion on these roads: The more cars use the roads, the more traffic holdups will arise. The aspect of rivalry is represented by the congestion function

$$G_s(\varepsilon, G) = G k^\varepsilon K^{-\varepsilon}, \quad 0 \leq \varepsilon \leq 1 \quad , \quad (2)$$

¹In the following parts of the paper time indices are suppressed.

²We abstract from technological progress. However, the structural results of the model would not be changed if we introduced labor augmenting technological progress.

where G specifies the total amount and G_s the actually available amount of the publicly provided input. The parameter $K = k \cdot L$ denotes aggregate capital while ε characterizes the degree of congestion of the public input G : If $\varepsilon = 0$ the governmental input is a pure public good, as e. g. the legal framework, whereas $\varepsilon = 1$ reflects the other polar case of proportional congestion, i. e. $G_s = G/L$.³ The road network displays partial rivalry, which is captured by $0 < \varepsilon < 1$ and leads to an individually available amount of roads $G/L < G_s < G$. Independent from the degree of congestion the individually available amount of G_s increases with a rise in G . With this specification of the congestion function the marginal product of private capital is given by $f_k \equiv \frac{df}{dk} = f_1 + f_2 \varepsilon \frac{G_s}{k} > 0$.

Private capital evolves over time as follows: It is reduced by depreciation where δ denotes the depreciation rate. At the same time capital increases as consequence of private investment I_i . Thus the net investment of private capital in each time increment is given by

$$\dot{K}_i = I_i - \delta K_i \quad . \quad (3)$$

We assume that private capital accumulation goes along with adjustment costs, ϕ , that depend positively upon the amount of investment and negatively on the available public input.⁴ The adjustment costs are specified in analogy to Barro and Sala-I-Martin (2004)

³This specification of the congestion function is borrowed from Edwards (1990). Aside from this there exist other specifications, e. g. Barro and Sala-I-Martin (2004) model congestion as relation between individual capital and aggregate production. Eicher and Turnovsky (2000) distinguish between absolute and relative congestion. Following their notation the formulation of congestion in (2) reflects the situation of relative congestion.

⁴The implications of labor and capital adjustment costs are discussed in detail in the existing literature (see e. g. Hamermesh and Pfann (1996) for an overview). Within the models analyzed there capital adjustment costs refer to the relation between investment and the existing capital stock. The impact of pro-

or Turnovsky (1996) by the function

$$\phi = \phi \left(\frac{I_i}{G_s} \right), \quad \phi' > 0 \quad . \quad (4)$$

Thus, the costs increase with the relation I_i/G_s . Due to $\phi' > 0$ adjustment costs $I_i\phi \left(\frac{I_i}{G_s} \right)$ are convex.

With these specifications of the production function (1), the congestion function (2) as well as with the formulation of the adjustment costs (4) it is now possible to identify the different channels that reflect the impact of the public production input within the model. Starting point of the analysis is the available amount of infrastructure, G_s , that is positively linked with total amount of G and negatively with the degree of congestion, ε .

The impact of the public production input within the model is twofold and includes:

- The *production effect*: Within production function (1) infrastructure is modeled as input that is complementary to private capital, $f_{k,G_s} > 0$. An increase of G_s not only enhances output directly but also acts indirectly via an increase in the marginal product of private capital. Additionally, prevailing congestion affects the productivity of private capital. These direct and indirect effects that act via the production function will be summarized by the term production effect.
- The *adjustment cost effect* is also subject to governmental activity: An increase in G_s reduces the relationship between private investment and governmental activity, I_i/G_s , and yields a decrease of the adjustment costs. This formulation might be motivated as follows: If G_s is interpreted as being the amount of infrastructure

ductive governmental activity within the adjustment process is discussed in the context of a growth model by Turnovsky (1996).

available to the individual firm then adjustment costs in the open countryside (low G) are higher than they are in areas that are already richly endowed with infrastructure (high G). Thus governmental activity might influence the levels of the individual adjustment costs. A second argument gains importance: The adjustment costs increase with the level of congestion. Regions with less congestion (low ε) provide a more productive infrastructure to the firms and with this the adjustment costs are reduced.

In the following part of the paper we refer to the term adjustment cost effect whenever we analyze effects that arise in the context of the adjustment costs. We use the term production effect to illustrate effects that influence capital productivity. It will be shown that both effects clearly influence the equilibrium dynamics. These aspects will be discussed in detail in Section 5

3 Private capital investment

The individual firm maximizes the present value of the sum of net cash flows between times 0 and infinity, discounted in accordance with the market rate of return, $r(t)$. The net cash flow in each period is paid out as dividends to the shareholders. We focus on firms that have a number of equity shares outstanding, and the value of these shares at time 0 is determined on a stock market to be the amount $V(0)$. Firms pay the wage rate, w , for each unit of labor whereas the public input is provided at no charge. We neglect adjustment costs associated with labor. The firm's objective is to choose L_i , I_i and K_i at each date to maximize⁵

⁵Since we focus on an atomistic firm it is assumed that the market interest rate is exogenous and constant.

$$V(0) = \int_0^{\infty} e^{-r \cdot t} [F(K_i, L_i, G_s) - w \cdot L_i - I_i \cdot (1 + \phi)] dt \quad (5)$$

subject to constraint (3) and an initial value of capital, $K_i(0)$. We analyze the optimization problem by setting up the current-value Hamiltonian

$$J = e^{-r \cdot t} \left\{ F(K_i, L_i, G_s) - w \cdot L_i - I_i \cdot \left(1 + \phi \left(\frac{I_i}{G_s} \right) \right) + q \cdot (I_i - \delta K_i) \right\} \quad (6)$$

where q is the shadow price associated with (3). The shadow price has the units of goods per unit of capital at time t and represents the current-value shadow price of installed capital in units of contemporaneous output. The maximization problem entails the first-order conditions

$$f - f_l \cdot k = w \quad (7a)$$

$$1 + \phi + L_i \cdot \left(\frac{\iota}{G_s} \right) \cdot \phi' = q \quad (7b)$$

$$\frac{\dot{q}}{q} - \delta + \frac{1}{q} \left[f_k + L_i \cdot \left(\frac{\iota}{G_s} \right)^2 \cdot \phi' \cdot \varepsilon \frac{G_s}{k} \right] = r \quad (7c)$$

Within the first-order conditions (7a)–(7c) we have used the intensive form of the production function, $f(\cdot)$, and have written capital and gross investment as quantities per unit of labor, $\iota \equiv \frac{I_i}{L_i}$. Adjustment costs enter the first-order conditions as follows.

The static efficiency condition (7a) determines the amount of labor and involves the usual result that an optimum requires the coincidence of marginal product of labor and wage rate. There are no adjustment costs associated with changes in labor input. Equation (7b) represents the second static efficiency condition and determines the investment decision of the individual firm. It indicates that the marginal revenues of an investment (measured

in terms of the shadow price q) must equal the marginal costs of the investment. Without adjustment costs output could be transferred to capital in a 1:1 ratio. Then the efficiency condition implied $q = 1$. But now the marginal costs include the adjustment costs and thus the revenues induced by the firm's investment must increase in order to achieve an optimum. Equation (7c) is a dynamic efficiency condition and determines the optimal capital accumulation. The brackets include the marginal product of private capital plus the changes in adjustment costs that are weighted with the extent of investment, $f_k - I_i \frac{\partial \phi}{\partial K_i}$. Note that the marginal adjustment costs decrease with a rise in K_i . This effect is due to the available infrastructure: G_s increases with K_i (see equation (2)) and thus the relation I_i/G_s decreases.⁶ Both, marginal product of capital and marginal adjustment costs are deflated by the cost of capital, q , and the shadow price exceeds unity the more the higher the adjustment costs are. An optimum requires that the market interest rate, r , coincides with the sum of (i) net rate of capital gain ($\frac{\dot{q}}{q} - \delta$) and (ii) marginal product of capital plus marginal adjustment costs that are weighted with the investment.

To fully specify the optimization problem the transversality condition must be met. It is given by

$$\lim_{t \rightarrow \infty} (q \cdot K_i \cdot e^{-rt}) = 0 \quad (8)$$

and implies that either the capital stock or its value must equal zero at the end of the optimization horizon.

⁶Formally this results from the relation $\frac{\partial \phi}{\partial K_i} = \frac{\partial \phi}{\partial I_i} \cdot \frac{\partial I_i}{\partial G_s} \cdot \frac{\partial G_s}{\partial K_i} = \phi' \cdot \left(-\frac{I_i}{G_s^2}\right) \cdot \varepsilon \frac{G_s}{K_i} < 0$.

4 Implications of the first-order conditions

The first-order conditions (7a)–(7c) illustrate that the public production input and its quality (measured by ε) influence the firm's investment decision in various ways. The following argumentation distinguishes between the production and the adjustment cost effect as discussed in Section 2.

Within equation (7a) only the production effect emerges: All inputs are complementary and hence the the marginal product of labor increases with G_s . Compared to a model without a public production input the optimal amount of labor increases whereas the adjustment costs are not affected.

Within the optimality condition (7b) only the adjustment cost effect plays a role: A rise in G_s reduces the adjustment costs, ϕ , and also the marginal adjustment costs as long as $\phi' > 0$. The opposite applies for a reduction of G_s that is accompanied by increasing costs. This reflects the fact that investment costs are lower if the factory area is already developed. If adjustment costs are absent ($\phi = \phi' = 0$) the marginal product of one unit capital investment measured in units of output equals the shadow price of capital, $q = 1$. In case of positive adjustment costs the shadow price of capital exceeds unity and it is needed to use more than one unit of output to get one unit of capital. Thus optimality of investment requires a higher marginal product of investment and the necessary surplus increases with ϕ . Additionally this effect is reinforced by positive marginal adjustment costs, $\phi' > 0$. The available infrastructure, G_s , and their determinants ε and G then become crucial: The lower ε and the higher G the higher is also G_s . A higher amount of available infrastructure reduces the height of the adjustment costs, reduces directly the relation I_i/G_s as well as ϕ' . One might conclude that more infrastructure increases the incentives of private investment due to a reduction of the adjustment costs.

In the context of the dynamic efficiency condition (7c) the production effect as well as the adjustment cost effect (indirectly via ϕ') arises. Since the available infrastructure, G_s , increases with the individual capital stock, K_i , the marginal adjustment costs decrease during the process of capital accumulation. Again we begin with the simple case without adjustment costs and thus a shadow price equal to $q = 1$. This implies $\frac{\dot{q}}{q} = 0$ and the dynamic efficiency condition reduces to the well known relation $f_k - \delta = r$: The net marginal product of capital must equal the interest rate. Then only the production effect of governmental activity emerges. If adjustment costs arise ($\phi > 0$) the degree of rivalry becomes crucial since ε influences the production effect and the adjustment cost effect. If congestion prevails ($\varepsilon > 0$) individual productivity of infrastructure depends on the degree of congestion as well as on the individual capital stock.⁷ As consequence the production effect is influenced. Aside from this congestion also has an impact on the extent of the adjustment cost effect: It vanishes if the public input is a pure public good ($\varepsilon = 0$) and the adjustment costs rise with an increase in congestion. To sum up: The extents of both the production and the adjustment cost effect are influenced by congestion.

5 Equilibrium and transitional dynamics

We now provide a formal analysis of the equilibrium and the corresponding transitional dynamics. To derive the equilibrium level of private capital it is necessary to specify the general adjustment cost function in equation (4). Although most of the empirical and theoretical studies analyze either convex or non-convex adjustment costs stylized facts about them do not exist. Actually the costs strongly vary between single industries (see e. g. Cooper and Haltiwanger (2003)). For our analysis we thus choose the most

⁷See e. g. Turnovsky (2000b) for a detailed discussion of this effect.

simple case in which the adjustment costs are proportional to the relation I_i/G_s and hence total adjustment costs $I_i\phi\left(\frac{I_i}{G_s}\right)$ are convex. With this specification we follow Turnovsky (1996). If one additionally assumes a positive interdependency between I_i/G_s and the shadow price, q , it is possible to derive the inverse relationship

$$\frac{I_i}{G_s} = \psi(q), \quad \psi' > 0 \quad . \quad (9)$$

Together with equations (4) and (11) the adjustment cost function can be specified as⁸

$$\phi\left(\frac{I_i}{G_s}\right) = b\frac{I_i}{G_s} = b \cdot \psi(q), \quad \phi' = b > 0 \quad . \quad (10)$$

The marginal adjustment costs are constant and b might be interpreted as sensitivity parameter to analyze the impact of the available infrastructure on ϕ . Together with efficiency condition (7b) optimal investment requires

$$\psi(q) = \frac{q-1}{2b} \quad . \quad (11)$$

We now present the discussion in terms of capital per unit of labor, $k = \frac{K_i}{L_i}$. The equilibrium is defined as a situation in which the firm's capital and the shadow price of capital are constant, $\dot{k} = 0$ and $\dot{q} = 0$. Since we focus on firms acting in competitive markets we assume an exogenous market interest rate, r . Formally, the equilibrium might be described by a system of differential equations that result from equations (3) and (7c) together with (9) and (10) as

⁸Aside from convex adjustment costs that most frequently are modeled as being quadratic there exist also non-quadratic costs. This is usually the case in the context of discrete costs (see e. g. Caballero (1999) for an overview or Caballero and Engel (1999), Cooper and Haltiwanger (2003) and Thomas (2001) for discrete adjustment costs within certain industries).

$$\dot{k} = \iota - \delta k \quad (12a)$$

$$\dot{q} = (r + \delta)q - \left[f_k + \frac{1}{L_i} \cdot b \cdot \psi^2 \cdot \varepsilon \frac{G_s}{k} \right] \quad (12b)$$

It can easily be shown that (12b) depends via ψ on the investment ratio, $\frac{\iota}{k}$, since from equation (9) follows

$$\Psi(q) = \frac{I_i}{G_s} = \frac{\iota}{k} \cdot \frac{L_i}{\frac{G_s}{k}} \quad (13)$$

Equalizing equations (11) and (13) illustrates that the shadow price of capital is also a function of the investment ratio. It is thus necessary to modify equation (12b) in order to derive the equilibrium. If we define the investment ratio as $\frac{\iota}{k} \equiv \mu$ and analogously $\left(\frac{\dot{\iota}}{k}\right) \equiv \dot{\mu}$ changes in the shadow price of capital, \dot{q} , in equation (12b) might be rewritten as⁹

$$\dot{\mu} = \frac{\frac{G_s}{k}}{2bL_i} [r + \delta - f_k] + \mu(r + \delta + (1 - \varepsilon)\delta) - \mu^2 \left(1 - \frac{1}{2}\varepsilon\right) \quad (14)$$

The equilibrium is characterized by a constant capital stock and a constant investment ratio.

With this it is possible to illustrate the equilibrium (k^*, μ^*) as intersection between the functions $\dot{k} = 0$ from (12a) and $\dot{\mu} = 0$ in (14) in a phase plane diagram $\mu(k)$ (see Figure 1). Individual capital as given by equation (12a) remains constant if investment ratio and depreciation rate coincide. This applies independent from the level of k and hence the

⁹The formal derivation of equation (14) can be found in Appendix A. Note that a small firm perceives the aggregate capital stock as constant and exogenous. Hence the growth rate of aggregate capital does not enter equation (14). The same argumentation holds with respect to extent and changes in infrastructure. Thus the growth rate of G does not enter (14) either.

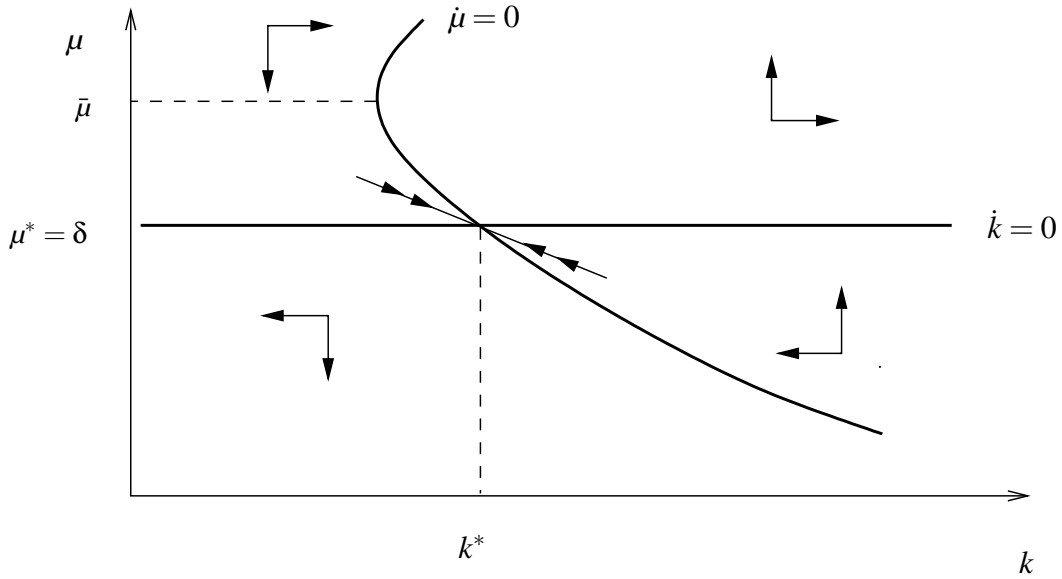


Figure 1: Equilibrium capital and investment

function $\dot{k} = 0$ is parallel to the abscissa at a constant investment ratio denoted by $\mu^* = \delta$. The slope of function $\dot{\mu} = 0$ might be derived via implicit differentiation of equation (14). It results¹⁰

$$\frac{d\mu}{dk} = -\frac{\frac{1}{2bL_i} \left[\frac{\partial G_s}{\partial k} (r + \delta - f_k) - \frac{G_s}{k} \frac{\partial f_k}{\partial k} \right]}{r + \delta + (1 - \varepsilon)\delta - 2\mu \left(1 - \frac{1}{2}\varepsilon\right)} \geq 0 \quad (15)$$

The sign of the numerator is unequivocally positive whereas the sign of the denominator is ambiguous and changes for a certain investment ratio $\bar{\mu} \equiv \delta + \frac{r}{2-\varepsilon} > \delta$: If $\mu < \bar{\mu}$ the denominator is positive whereas it is negative if $\mu > \bar{\mu}$. Together with the negative sign in equation (15) function $\dot{\mu} = 0$ might be illustrated as presented within Figure 1. It results the unique equilibrium (k^*, μ^*) that is determined by the intersection of both functions.

We now focus on the transitional dynamics of the system and on stability aspects. Since the equilibrium investment ratio is given by $\mu^* < \delta$ the following argumentation is carried

¹⁰See Appendix B for the derivation and a proof of the signs.

out for parameters $\mu < \bar{\mu}$. Using equation (12a) it becomes obvious that the capital stock increases (decreases) if the initial investment ratio exceeds (is smaller than) the depreciation rate, $\delta = \mu^*$. The horizontal arrows within Figure 1 illustrate this argument. Using equation (14) the investment ratio increases (decreases) if it is initially right (left) to function $\dot{\mu} = 0$. This is illustrated by the vertical arrows. All together the system is saddle point stable. Initial combinations of capital and investment ratio that lie on the stable path lead to the equilibrium. For any initial capital stock, $k(0)$, the individuals will choose an investment ratio so that both, k and μ , are lying on the stable path and thus the equilibrium will be reached. Note that existence, uniqueness and stability characteristics of the system as well as the optimal investment ratio are independent of the extent and the degree of congestion. However, the equilibrium capital, k^* , depends on the economy's endowment of infrastructure, G , and is also affected by congestion. The details are discussed within Sections 6 and 7.

6 Public inputs and equilibrium

The equilibrium investment ratio, μ^* , has been derived from equation (12a). It only depends on the depreciation rate, δ , and is independent from the public input. In contrast to this the equilibrium capital, k^* , is determined by the individually available amount of the public input, G_s . The following discussion demonstrates the importance to distinguish between the two parameters of the public input, G and ε . In case of a constant investment ratio and for an exogenously given level of L_i the relationship G_s/k determines also I_i/G_s and with this the level of the adjustment cost and the investment ratio (see equation (9)). Hence we refer to the adjustment cost effect (denoted by *ACE* within the equations) whenever changes in the public input affect the term G_s/k . Analogously we

refer to the production effect (denoted by *PE*) whenever the marginal product of capital, f_k , is affected. It will be shown that a third effect arises that is denoted by level effect (*LE* in equation (19)).

(i) The impact of G : We begin the analysis with a discussion of the positive impact of governmental expenditures on the equilibrium capital stock. Starting point is the initial equilibrium capital, k_0^* , in Figure 2. Formally the impact of an increase of infrastructure on equilibrium capital might be reduced to the relationship

$$\text{sign } \frac{dk}{dG} = - \text{sign } \frac{\partial \dot{\mu}}{\partial G_s} . \quad (16)$$

The sign of $\frac{\partial \dot{\mu}}{\partial G_s}$ is unequivocally negative as illustrated within

$$\frac{\partial \dot{\mu}}{\partial G_s} = \frac{1}{2bL_i} \left[\underbrace{\frac{\partial \frac{G_s}{k}}{\partial G_s}}_{>0} \underbrace{(r + \delta - f_k)}_{\leq 0} - \underbrace{\frac{G_s}{k}}_{>0} \underbrace{f_{k,G_s}}_{>0} \right] < 0 . \quad (17)$$

ACE *PE*

Introducing equation (17) in equation (16) it can be shown that the equilibrium capital stock unequivocally increases with a rise of G . Both effects account for that result: An increase in G also rises the individually available amount of infrastructure, G_s , and reduces the adjustment costs. In addition, due to the complementarity of public and private inputs, $f_{k,G_s} > 0$, the productivity of private capital increases. Thus capital investment becomes more attractive and k^* unequivocally increases. This result is illustrated by the transition of the initial equilibrium, k_0^* , to the new equilibrium, k_1^* , within Figure 2.

(ii) The impact of ε : In contrast to the amount of infrastructure the effect of different degrees of rivalry on the equilibrium capital stock is not clear. We show that an increase in rivalry which reduces the individually available amount of the public input, may even increase the investment in the physical capital stock. Again the formal analysis may be reduced to the simple relationship

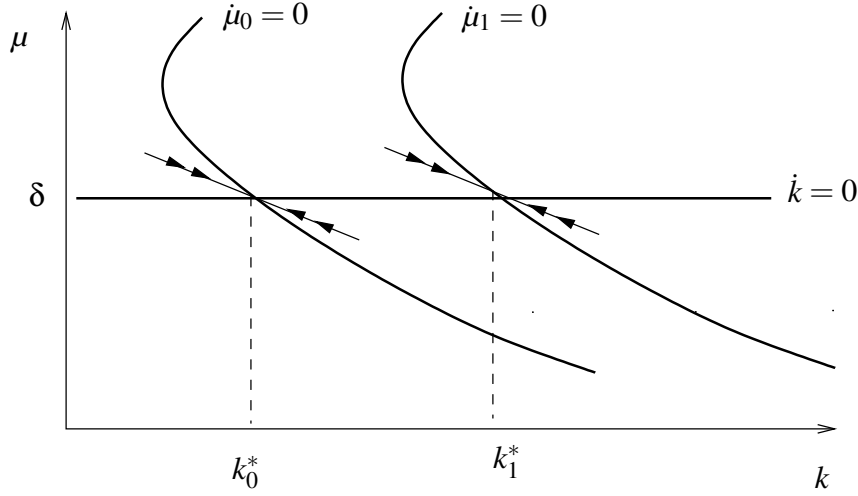


Figure 2: Impact of governmental expenditures, G_s , on the capital stock, k^*

$$\text{sign} \frac{dk^*}{d\varepsilon} = -\text{sign} \frac{\partial \dot{\mu}}{\partial \varepsilon} . \quad (18)$$

It can be shown that the sign of $\frac{\partial \dot{\mu}}{\partial \varepsilon}$ depends on the extent of the production and the adjustment cost effects. Again the different effects may be derived from equation (14).

$$\frac{\partial \dot{\mu}}{\partial \varepsilon} = \frac{1}{2bL_i} \left[\underbrace{\frac{\partial \frac{G_s}{k}}{\partial \varepsilon} (r + \delta - f_k)}_{ACE \leq 0} - \underbrace{\frac{G_s}{k} \frac{\partial f_k}{\partial \varepsilon}}_{PE > 0, \geq 0} \right] - \underbrace{\mu \left(\delta - \frac{1}{2} \mu \right)}_{LE > 0} \gtrless 0 . \quad (19)$$

Using equations (18) and (19) it becomes obvious that the adjustment cost effect decreases the equilibrium capital whereas the production effect increases k^* . Additionally another positive effect –in the following called level effect– arises. It is positive and its extent depends on the level of the investment ratio, μ , that in equilibrium equals δ .

The economic intuition for these effects may be summarized as follows: An increase in rivalry reduces the available infrastructure, increases the adjustment costs $\phi \left(\frac{I_t}{G_s} \right)$ and capital accumulation becomes less attractive. In contrast to this higher rivalry implies that

the individually perceived marginal product of private capital increases. This stimulates capital accumulation and k^* increases and is illustrated in the context of Figure 3.¹¹

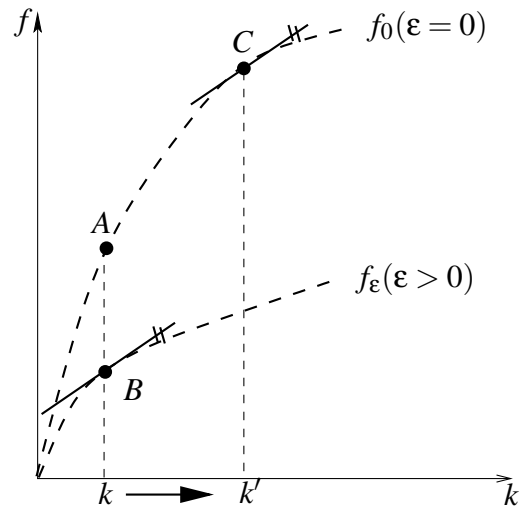


Figure 3: The production effect of congestion

The figure covers two production functions, one that reflects the public input as pure public good, $f_0(\epsilon = 0)$, and another in which congestion arises, $f_\epsilon(\epsilon > 0)$. Then productivity of capital is reduced and f_0 is scaled down to f_ϵ . If the firm's capital stock is given by k and if congestion arises the actual marginal product of capital is given by the slope of f_ϵ in point B . In contrast to this the individually perceived marginal product is higher and is represented by the slope of f_0 in point A . Thus if optimization requires a marginal

¹¹Note that behind the perceived marginal product of capital lies an external effect, since actually capital productivity is reduced if congestion arises. Hence an increase of equilibrium capital as consequence of an increase in rivalry is not optimal. The welfare economic implications may be analyzed in the context of an aggregate model. They include the financing restrictions that must be met if the governmental input is provided without user fees (see e. g. Barro (1990)) or Ott and Turnovsky (2006) for a recent discussion of this aspect in the context of partially excludable public inputs.

product to be equal to the slope in point B then capital accumulation takes place until k' is reached since the individuals perceive f_ε as the relevant production function. In point C the marginal product coincides with the one in point B . Hence, the production effect implies $k' > k$ and the gap between both increases with rising congestion. Note that the level effect also arises as consequence of congestion and stimulates capital accumulation. The economic intuition for this is that the individuals perceive the public input as pure public good – in fact for all degrees of congestion. They thus assume production function f_0 in Figure 3 to be relevant for their investment decision. Putting the three effects together it becomes obvious that the total effect of an increase in rivalry on the equilibrium capital stock is not clear: The adjustment cost effect is negative whereas production and level effect are positive. Hence whether the equilibrium capital increases or decreases depends on the extents of the effects. However, in both cases – changes of ε or G – the characteristics of the equilibrium are not affected: There results a unique equilibrium that is saddle point stable.

7 Policy implications

While in the last Section 6 the objective was to analyze the general implications of the public input for the resulting equilibrium capital stock we now focus on the policy implications of the model. Since an increase in G unequivocally spurs private capital investment the government can stimulate accumulation via an increase of the amount of the publicly provided input. Usually the government must meet certain financing restrictions with respect to its budget but to address these aspects a closed model is required. We abstract therefrom within this paper. Due to the unclear total effect the policy implications of the degree of congestion are more sophisticated. Turnovsky (1996, p. 363) argues that

'... the degree of congestion is to some extent the outcome of a policy decision, and once determined, the degree of congestion turns out to be a critical determinant of optimal tax policy.' The prevailing degree of rivalry might be interpreted as being the result of institutional arrangements: A ban of driving for trucks on certain routes or the implementation of road user fees may decrease the volume of traffic thus reducing the degree of congestion. The same result may be achieved as consequence of high gasoline prices that may be the result of governmental activity (e. g. tax increases on gasoline).¹² In the latter case changes in the degree of rivalry are the outcome of fiscal policy.

To be more precise about the impact of different degrees of rivalry on the individual investment decision it is helpful to specify output per capita in equation (1) as CES production function

$$y = f(k, G_s) = A[\alpha k^{-\rho} + (1 - \alpha)G_s^{-\rho}]^{-\frac{1}{\rho}}, \quad A > 0, \quad 0 < \alpha < 1, \quad -1 < \rho \neq 1 \quad (20)$$

and to calibrate the equilibrium capital stock for alternative parameter constellations. The parameter $\sigma \equiv \frac{1}{1+\rho}$ denotes the elasticity of substitution between the two inputs.

We focus on different degrees of congestion and distinguish situations in which (i) private and public input are highly ($\rho = -0.5$) or slightly ($\rho = 5$) substitutable and (ii) in which governmental expenditure per capita are relatively high ($G/L = 1000$) or relatively low ($G/L = 100$). The other parameters are specified as denoted within Table 1 which summarizes the resulting equilibrium capital stocks.

Taking a look at the resulting values it becomes clear that in case of relatively low governmental expenditure per capita ($G/L = 100$) a reduction of rivalry stimulates private

¹²We abstract from the arguments of increasing gas prices that arise due to resource scarcity or market distortions.

	$\frac{G}{L} = 1000$		$\frac{G}{L} = 100$	
	$\rho = -0.5 (\sigma = 2)$	$\rho = 5 (\sigma = .167)$	$\rho = -0.5 (\sigma = 2)$	$\rho = 5 (\sigma = .167)$
$\varepsilon = 0$	12795.6	1303.4	1754.9	1136.8
$\varepsilon = 0.25$	13984.2	1466.5	1128.8	689.2
$\varepsilon = 0.5$	15222.6	1796.0	714.6	431.0
$\varepsilon = 0.75$	16521.9	2399.4	447.6	282.8
$\varepsilon = 1$	17896.7	3108.8	278.2	191.4

Table 1: Equilibrium capital for the CES production function

$\alpha = 0.75, r = 0.01, \delta = 0.05, \mu = 0.05, A = 0.4, b = 0.5, G = 1000$

capital accumulation. The adjustment cost effect dominates both the production and the level effect. Hence, the reduction of congestion spurs investment and with this leads to an increase in k^* . The level of the substitution parameter ρ does not influence the direction of this total effect but its extent. If the government wishes to increase private investment it may thus either increase G (whereupon any financial restrictions have to be regarded) or make institutional arrangements to reduce congestion (e. g. certain bans on driving or user fees). It is also possible to include both instruments in a policy mix and to use the revenues that result out of the institutional arrangements in order to finance the provision of the public input and thus to reinforce the stimulating effect.

The contrary results if the governmental expenditure per capita are relatively high ($G/L = 1000$). Then any institutional arrangements that reduce congestion also decrease the equilibrium capital stock since production and level effect dominate the adjustment cost effect. The decrease in congestion induces an increase in the productivity of the public input. Hence, there is a crowding out of physical capital. In a situation with relatively ample governmental expenditures, the crowding out effect dominates and thereby leads

to a decrease in private investment.

It is possible to sum up the policy implications of the model: If the government pursues the goal of stimulating private capital accumulation it may basically choose between fiscal and/or institutional instruments. Institutional arrangements that reduce the degree of congestion may even reduce capital accumulation (e. g. if G/L is relatively high) whereas an increase in the amount of the public input unequivocally spurs investment. The argumentation makes also clear that if a government enhances its expenditure in order to stimulate private investment it must also take care not to exceed a certain critical relation G/L because maybe then the possibility to reduce ϵ in order to stimulate capital ceases to exist. This emphasizes the argumentation that if the government is becoming too big this may restrict its own possibilities of action.

8 Conclusions

Recent discussions stress the point that firm specific aspects are not the unique determinant of capital adjustment. Aside from them also the economic environment like the provision of productive governmental inputs or the degree of congestion gain importance. This argument is the starting point of the paper: We analyze within an dynamic model how a firm decides on capital investment if the accompanying adjustment costs are a function of governmental activity. Following Aschauer (1989b), Barro (1990) or Turnovsky (2000b) we interpret this governmental activity as being congested infrastructure and assume that this affects the firm's capital adjustment costs. Governmental activity consists of fiscal and/or institutional policies. The former refers to the provision of a certain amount of the public input whereas the latter applies to the prevailing degree of congestion. We analyze how the equilibrium capital stock is changed if either the amount of

infrastructure or the degree of rivalry are changed. It is possible to identify three effects of the public input within the model: a production, an adjustment cost and a level effect.

The main results may be summarized as follows: While an increase in the amount of the public input unequivocally spurs private capital investment and increases the equilibrium capital stock, the result becomes ambiguous with respect to changes in the degree of rivalry. Then the direction of the production effect on the one hand and the adjustment cost as well as the level effect on the other hand differ. The total effect of changes in the degree of rivalry then depends on the dominating effect(s). In case of a CES production function it is shown that the production and the level effect dominate if the governmental expenditure per capita are relatively high. Then an increase in congestion leads to a higher equilibrium capital stock since the individuals perceive capital as being more productive. The opposite applies if governmental expenditure per capita are relatively low. Then the adjustment cost effect dominates and the equilibrium capital stock decreases. Thus, governmental activity that wishes to reduce congestion may even end up in a reduction of private investment.

Appendix

A: Derivation of (14)

Rearranging (11) yields $q = 1 + 2b\psi \implies \dot{q} = 2b\dot{\psi}$. Thus (12b) results as

$$2b\dot{\psi} = (r + \delta)(1 + 2b\psi) - \left[f_1 + \left(f_2 + \frac{1}{L_i} \cdot b\psi^2 \right) \cdot \varepsilon \frac{G_s}{k} \right] \quad (21)$$

From the definition of $\psi = \frac{I_i}{G_s}$ in (9) follows

$$\begin{aligned} \dot{\psi} &= \frac{\dot{I}_i}{G_s} - \frac{I_i}{G_s} \cdot \frac{\dot{G}}{G} && \text{where } i = \frac{\partial \frac{I_i}{L_i}}{\partial t} = \frac{\dot{I}_i}{L_i} \\ &= L_i \cdot \frac{i}{G_s} - L_i \cdot \frac{I_i}{G_s} \left[\frac{\dot{G}}{G} + \varepsilon \frac{\dot{k}}{k} - \varepsilon \frac{\dot{K}}{K} \right] && \text{where } \left(\frac{i}{k} \right) = \frac{i}{k} - \frac{l}{k} \cdot \frac{\dot{k}}{k} \\ &= L_i \cdot \frac{k}{G_s} \left[\frac{i}{k} - \frac{l}{k} \cdot \frac{\dot{k}}{k} + \frac{l}{k} \cdot \frac{\dot{k}}{k} - \frac{l}{k} \cdot \frac{\dot{G}}{G} - \varepsilon \cdot \frac{l}{k} \cdot \frac{\dot{k}}{k} + \varepsilon \frac{l}{k} \cdot \frac{\dot{K}}{K} \right] \\ \dot{\psi} &= L_i \cdot \frac{k}{G_s} \left[\left(\frac{i}{k} \right) + (1 - \varepsilon) \frac{l}{k} \cdot \frac{\dot{k}}{k} - \frac{l}{k} \cdot \frac{\dot{G}}{G} + \varepsilon \frac{l}{k} \cdot \frac{\dot{K}}{K} \right] \end{aligned} \quad (22)$$

Introducing (22) in (21) and solving for $\left(\frac{i}{k} \right)$ yields

$$\begin{aligned} \left(\frac{i}{k} \right) &= \frac{(r + \delta)(1 + 2b\psi) \frac{G_s}{k} - \frac{f_1 G_s}{2bL_i} - \frac{f_2 \varepsilon \left(\frac{G_s}{k} \right)^2 - \psi^2 \varepsilon \left(\frac{G_s}{k} \right)^2}{2bL_i} \\ &\quad - \frac{l}{k} \left((1 - \varepsilon) \frac{\dot{k}}{k} - \frac{\dot{G}}{G} + \varepsilon \frac{\dot{K}}{K} \right) && \text{with } \psi = \frac{I_i}{G_s} \\ \left(\frac{i}{k} \right) &= \frac{(r + \delta)(1 + 2b \frac{I_i}{G_s}) \frac{G_s}{k} - \frac{f_1 G_s}{2bL_i} - \frac{f_2 \varepsilon \left(\frac{G_s}{k} \right)^2}{2bL_i} - \frac{1}{2} \varepsilon \left(\frac{l}{k} \right)^2}{2bL_i} \\ &\quad - \frac{l}{k} \left((1 - \varepsilon) \frac{\dot{k}}{k} - \frac{\dot{G}}{G} + \varepsilon \frac{\dot{K}}{K} \right) \end{aligned} \quad (23)$$

An atomistic firm interprets the aggregate capital stock as well as the amount of G as being exogenous and constant. With this it follows that $\frac{\dot{K}}{K} = \frac{\dot{G}}{G} = 0$. The growth rate of capital per capita results from (3) as $\frac{\dot{k}}{k} = \frac{i}{k} - \delta$. Together with $\left(\frac{i}{k}\right) = \dot{\mu}$, $\frac{i}{k} \equiv \mu$ and rearranging $\dot{\mu}$ equation (14) results.

B: Analysis of the sign of (15)

The slope of function $\dot{\mu} = 0$ from (14) is derived via implicit differentiation: $\frac{d\mu}{dk} = -\frac{\frac{\partial \dot{\mu}}{\partial k}}{\frac{\partial \dot{\mu}}{\partial \mu}}$.

Taking the partial derivatives it follows

$$\frac{\partial \dot{\mu}}{\partial k} = \frac{1}{2bL_i} \left[\frac{\partial \frac{G_s}{k}}{\partial k} (r + \delta - f_k) - \frac{G_s}{k} \frac{\partial f_k}{\partial k} \right] \quad (24a)$$

$$\frac{\partial \dot{\mu}}{\partial \mu} = r + \delta + (1 - \varepsilon)\delta + \frac{\dot{G}}{G} - 2\mu \left(1 - \frac{1}{2}\varepsilon \right) \begin{matrix} \geq 0 \\ \leq 0 \end{matrix} \iff \mu \begin{matrix} \leq \\ \geq \end{matrix} \bar{\mu} \quad (24b)$$

Equation (24a) is unequivocally positive and includes

$$\frac{\partial \frac{G_s}{k}}{\partial k} = (\varepsilon - 1) \frac{G_s}{k^2} < 0 \quad (25a)$$

$$\frac{\partial f_k}{\partial k} = f_{11} - \varepsilon \frac{G_s}{k} \left(\frac{f_2}{k} - f_{21} \right) < 0 \quad (25b)$$

In contrast to this the sign of (24b) changes at a certain investment ratio $\bar{\mu} \equiv \delta + \frac{r}{2-\varepsilon} > \delta$:

$$\frac{\partial \dot{\mu}}{\partial \mu} \begin{matrix} \geq 0 \\ \leq 0 \end{matrix} \iff r + \delta + (1 - \varepsilon)\delta + \frac{\dot{G}}{G} - 2\mu \left(1 - \frac{1}{2}\varepsilon \right) \begin{matrix} \geq 0 \\ \leq 0 \end{matrix} \quad (26a)$$

$$\iff \bar{\mu} \equiv \delta + \frac{r + \frac{\dot{G}}{G}}{2 - \varepsilon} \begin{matrix} \geq \\ \leq \end{matrix} \mu \quad (26b)$$

If $\mu > \bar{\mu}$ the sign of (26b) becomes negative; analogously a positive relation holds if $\mu < \bar{\mu}$.

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