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Growth and Welfare Effects of Tax Cuts: The Case of a Productive Public Input with Technological Risk

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Abstract. This paper analyzes the dynamic impact of tax cuts within a stochastic model of endogenous growth with a congested public input. A decreasing taxation of deterministic income parts leads to the well-known positive growth effect. Nevertheless, due to the insurance effect associated with the taxation of stochastic income flows, the overall growth impact of taxation is ambiguous. It is shown that the optimal structure of financing government expenditure does not only depend on the degree of rivalry but also on the degree of risk aversion. The optimal real value of government debt decreases with a rise in congestion. We identify that in the case of proportional congestion, the base for tax cuts should be the growth neutral consumption tax. Maximizing the growth rate does not automatically coincide with maximizing welfare. Hence, the base for tax cuts gains importance to realize a welfare optimal policy.

Keywords: Tax cuts, congestion, uncertainty, growth

JEL codes: D8, D9, H2, O4

I. Introduction

Due to the persistent economic slowdown in most western countries, there is a broad political discussion about tax cuts in order to stimulate growth. Within deterministic growth models, a reduction of the income tax rate increases the after tax capital return, thus inducing a higher level of capital accumulation and the growth rate increases. In models with a tax financed public production input, the growth effect of tax cuts is ambiguous and crucially depends on the chosen tax base. Due to a reduction of income taxes, positive growth effects only apply if the tax rate was initially suboptimally

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high (see, for example, Barro, 1990). Another argument in favor of tax cuts draws on Laffer (1979) and identifies them as a possible source of an increase in governmental revenues. If these revenues are used as production input, a higher amount of the public input ceteris paribus goes along with an increase in productivity of the private factors. Thus, tax cuts that enhance total governmental revenues stimulate growth. Bruce and Turnovsky (1999) discuss the condition in which intertemporal Laffer-curve effects obtain. The expected welfare effects of tax cuts in a dynamic context may even justify an increase in government debt.

Concerning the growth effects of income taxation stochastic growth models highlight the fact that the impact of fiscal policy crucially depends on the assumptions about risk (see, for example, Eaton, 1981; Smith, 1996; Clemens and Soretz, 1997, 2004; Corsetti, 1997; Turnovsky, 1999, 2000). Aggregate income risk influences the macroeconomic equilibrium and with this the efficacy of fiscal policy in various ways. The growth effects of changes in income taxation are ambiguous and tax cuts might even lead to a reduction of the growth rate. On the contrary, the introduction of government bonds or changes in lump-sum taxation is growth neutral within the usual settings and cuts of these taxes would not stimulate growth.

Striving for a reduction in taxes to induce growth effects implicitly assumes that welfare is enhanced with an increase in the growth rate. This is not always true as shown, for example, in those dynamic models in which the market equilibrium growth rate is suboptimally high. These arguments are discussed in the technical progress models that stand in the line of those from Grossman and Helpman (1991) or Aghion and Howitt (1992). With respect to governmental activity, suboptimally high growth rates also arise whenever the governmental input is subject to congestion (see, for example, Barro and Sala-I-Martin, 1992; Futagami et al., 1993; Fisher and Turnovsky, 1998; Turnovsky, 1999, 2000; Ott, 2001). The reason for that is a negative external effect of capital accumulation. In this case, reducing the income tax would reduce welfare because of the increased growth rate. Hence the welfare effects of income tax cuts also crucially depend on the characteristics of the governmental input.

Within the recent discussion on tax cuts, the argument mainly focuses on the growth effects induced by the enhanced after tax revenue of capital. From the point of view of welfare economics, this argumentation is too narrow as any fiscal policy is assessed via the induced growth and not the corresponding welfare effect. An optimal policy should, however, determine the tax rates in order to maximize welfare. If the production technology allows for congestion effects, excessive growth as a consequence of individual decisions is possible. Then, yet another growth enhancement due to a reduction of the income tax rate is accompanied by welfare losses and in spite of their growth-stimulating effects the government should refuse the corresponding tax cuts.

Furthermore, another argument also gains importance. While the government has various possibilities at its disposal to generate revenues, tax cuts need to be discussed with respect to alternative tax bases, mainly with respect to those instruments that are non-distortionary.

This paper picks up the arguments mentioned above and introduces them in a dynamic model in which the production technology involves private capital as well as a publicly provided input that might be congested. We adopt a congestion function that is based on Edwards (1990), for example, and is widely used in the public goods literature. With this assumption, the degree of congestion influences the marginal return of capital and hence individual capital accumulation. Due to the implementation of technological risk, individuals face an income uncertainty. In this context it can be shown that, on the one hand, reducing the income tax does not automatically increase the growth rate and that, on the other hand, an increase of the growth rate might be accompanied by welfare losses. Including the aspects of congestion and technological uncertainty, this paper is closely related to the paper of Turnovsky (1999). The main difference between Turnovsky (1999) and our model is the introduction of government debt. As a result, tax cuts do not necessarily imply a reduction in the total amount of governmental expenditures and the corresponding welfare effects have to be reconsidered. Thus, the government should not strive for tax cuts as an end in itself but focus on the welfare effects of any fiscal policy.

We consider three different policy parameters to finance the provision of the governmental input: taxes on income and consumption and governmental bonds. The main results of the paper can be summarized as follows: (i) Optimal income tax rates, along with the level of the non-distortionary fiscal instrument to balance the governmental budget, are influenced by uncertainty as well as by congestion. (ii) A cut of the tax levied on the deterministic income component enhances growth whereas the opposite applies to a reduction of the tax levied on the stochastic income component. Consequently, for the induced growth effect the assumption to which tax rate the tax cut applies is crucial. (iii) In case of uniform income taxation, the growth effect of tax cuts depends on the degree of risk aversion. (iv) The benchmark to assess tax cuts should not be the realized level of the growth rate but the corresponding welfare implications. As long as congestion arises, a decrease in the income tax rate might or might not increase welfare. The result depends on the optimal level and the actual value of the income tax rate relative to its optimum. The tax should not fall short of its optimal level.

The structure of the paper is as follows: After describing the assumptions of the model in Section II, we derive the social optimum in part III. Section IV analyzes the market equilibrium. In Section V changes in the market equilibrium growth rate that result from alternative tax policies are discussed and the fiscal instruments that allow for optimal financing are determined. The paper closes with a short summary.

II. The Model

The starting point of the analysis is a model of endogenous growth in which N identical individuals face a production risk. Hence, they maximize expected lifetime utility as given by

$$U = E_0\left[\int_0^{\infty} \frac{c^{1-\rho}}{1-\rho}\exp(-\beta t)\,dt\right], \quad \rho > 0, \ \rho \neq 1, \ \beta > 0. \tag{1}$$

The mathematical expectation, conditional on time 0 information, is denoted by E_0, $c(t)$ is time t consumption, $\exp(-\beta t)$ represents the discount factor with the instantaneous rate of time preference, β. The parameter ρ denotes the measure of relative risk aversion and equals the reciprocal of the intertemporal elasticity of substitution.[1] If the degree of relative risk aversion equals unity, instantaneous utility is logarithmic.

Each firm produces the homogeneous good, y, using capital as well as a productive governmental input according to the individual stochastic production function

$$y = \alpha\left(\frac{g}{k}\right)k(dt + \sigma dz), \quad \alpha' > 0, \ \alpha'' < 0, \ \sigma > 0. \tag{2}$$

In this equation, α may be interpreted as the productivity function. The productivity of capital depends on the average amount of public input, g, per unit of private capital, k. As we allow for rivalry in the use of the public input, capital productivity is determined by the proportional value of governmental expenditure. A rise in public input ceteris paribus increases individual capital productivity whereas an increase in capital reduces productivity, α. The productivity function, α, is assumed to satisfy the Inada-conditions. Since a feasible equilibrium requires a constant production elasticity of the governmental input, η, one could also assume the special case $\alpha = (g/k)^{\eta}$ for the productivity function.

Production per capita, y, depends on individual capital, k, as well as on productivity, α. For a constant relation of g/k, the production function is linear in capital, thus allowing for ongoing growth. The individuals face uncertainty as in each time increment, production is affected by a Hicks-neutral technological disturbance. They are serially uncorrelated, hence the Wiener process $dz(t)$ is a continuous Markov process with $dz \sim N(0, dt)$. For simplicity, depreciation is neglected and labor-leisure choice is not considered.

Congestion effects are introduced into the analysis via the parameter g that reflects the individually available amount of the governmental input. This formulation is borrowed from Edwards (1990) and adapted to endogenous growth models, for example by Futagami et al. (1993), Glomm and Ravikumar (1994, 1997) or Turnovsky (1999). The availability to the individual of the public input may thus be expressed by the congestion function

$$g = Gk^{1-\varepsilon}K^{\varepsilon-1}, \tag{3}$$

where $K \equiv Nk$ denotes the aggregate stock of capital, G equals the total amount of the public input and ε reflects the degree of congestion. The absence of any congestion is represented by $\varepsilon = 1$ in which case the public input is fully available to the representative agent, $g = G$. The other polar case, $\varepsilon = 0$, corresponds to proportional congestion and the total amount of the public input is divided by all N individuals. Hence, the individually available amount of the governmental input is reduced to $g = G/N$. For all intermediate cases, $0 < \varepsilon < 1$, the public input is characterized by partial congestion and $G > g > G/N$.

The congestion function (3) influences the level of α within the production function (2). A rising congestion reduces the individually available amount of the public input. The relation g/k, and hence individual productivity, declines. Just to replicate the congestion argument, it would be sufficient to use the congestion function as given in Equation (5) in the social planner's optimization problem. The reason why individual as well as aggregate capital enter into the congestion function originates in the negative externality associated with congestion. If congestion arises ($\varepsilon < 1$), the individuals are only aware of a part of their influence on the availability of the public input. Hence, they overestimate the availability of the public input.

To finance the provision of the public input, G, the government may use various instruments with different growth effects: The government may levy a linear tax on consumption ϖ as well as a proportional income tax consisting of two parts, τ and τ'. The income tax rates τ, τ' are set separately, as in Eaton (1981), in order to disentangle the effects of taxation of deterministic and random income parts. Since public revenues from income tax are stochastic whereas public expenditure G/N is assumed to be instantaneously deterministic, a further financing instrument is needed in order to balance the governmental budget in each time increment. Therefore, it is assumed that the government may finance deficits by issuing bonds. Consequently, the real value of these bonds, $b(t)$, is stochastic. It is measured in units of output and the bonds are characterized as perpetuities paying an uncertain real return. The expected rate of return of the bonds is given by i and the stochastic process of bond return is dz_b. The value of government bonds evolves according to

$$db = \left[\frac{G}{N} + ib - \tau\alpha k - \varpi c\right]dt + bdz_b - \tau'\alpha k\sigma dz. \tag{4}$$

III. First-Best Optimum

In this section the welfare maximizing growth rate, φ^*, together with the optimal ratio of government expenditure, $(g/k)^*$, and the propensity to consume, μ^*, will be derived. They will serve as benchmark in order to assess the decentral choices analyzed within the next section.

Consider a benevolent social planner who maximizes utility given in Equation (1) while taking the negative externality of capital accumulation into account. The congestion function in this case is obtained by setting $K = Nk$ in Equation (3) and becomes

$$g = GN^{\varepsilon-1}. \tag{5}$$

The optimization problem of the social planner may be written as

$$\max_{c,k,G} E_0\left[\int_0^\infty \frac{c^{1-\rho}}{1-\rho}\exp(-\beta t)\, dt\right] \tag{6}$$

$$\text{s.t. } dk = \left(\alpha k - c - \frac{G}{N}\right)dt + \alpha k\sigma dz. \tag{7}$$

Employing Itô's Lemma, the stochastic Bellman equation evolves to

$$B = \exp(-\beta t)\frac{c^{1-\rho}}{1-\rho} - \beta\exp(-\beta t)J(k) + \exp(-\beta t)J'(k)\left(\alpha k - c - \frac{G}{N}\right) + \frac{1}{2}\exp(-\beta t)J''(k)\sigma_k^2, \tag{8}$$

where the value function represents the maximum level of lifetime utility and is assumed to be of the time-separable form $\exp(-\beta t)J(k(t))$. The variance of capital is given by $\sigma_k^2 = E[dz_k^2]/dt$.

Maximizing the Bellman Equation (8) with regard to c and k leads to the necessary conditions of the optimization problem

$$c^{-\rho} = J'(k), \tag{9}$$

$$J'(k)(\alpha(1-\eta) - \beta) + \frac{1}{2}J'''(k)\sigma_k^2 + J''(k)(\alpha k - c + \alpha^2(1-\eta)k\sigma^2) = 0, \tag{10}$$

where ε denotes the production elasticity of the public input and is given by

$$\eta \equiv \frac{\partial y}{\partial k}\frac{G}{\alpha} = \frac{\alpha'}{\alpha}\frac{g}{k}. \tag{11}$$

Equation (9) comprises the usual result of intertemporal optimization, namely that the marginal utility of consumption corresponds to the (weighted) first derivative of the value function and is equalized across time. It determines the accumulation process together with (10). The optimal time paths for consumption and capital are functions of the derivatives of the value function and form a stochastic differential equation in $J(k)$. Hence, a maximum of utility as given by the integral (1) is obtained by determining a function $J(k)$ that solves the first-order conditions.

In addition to consumption and capital, the benevolent social planner decides on the optimal amount of government expenditure. The necessary condition results in

$$\alpha'(g/k)^* = \frac{N^{-\varepsilon}}{1 - \rho\alpha\sigma^2}, \tag{12}$$

which determines the optimal relation g/k by equating the marginal product and marginal cost of government expenditure. Together with Equation (11) and the congestion function of the social planner (5), the optimal production elasticity of the governmental input, η^*, may be derived

$$\eta^* = \frac{G}{(1 - \rho\alpha\sigma^2)N\bar{y}}, \tag{13}$$

with $\bar{y} \equiv E[y]/dt$ denoting the expected income divided by dt. It is influenced by congestion via individual production. The denominator in Equations (12) and (13) is of the same sign as the certainty equivalent of the stochastic capital return.[2] As the marginal productivity α' is assumed to be positive, the certainty equivalent has to be positive to ensure feasible solutions. Due to the assumption of diminishing returns, $\alpha'' < 0$, the underlying productivity shock reduces the optimal ratio $(g/k)^*$. Under uncertainty, governmental activity has two impacts: it enhances capital productivity and additionally increases the volatility of capital return given by σ_k^2. As long as the certainty equivalent of capital return is positive, the second (negative) effect of a rise in government expenditure does not overcompensate for the first (positive) effect. As a consequence, in a society of risk averse agents, the optimal level of government expenditure decreases with a rise in uncertainty.

In order to ensure feasible intertemporal consumption paths, the expected utility must be bounded. Thus, the following transversality condition has to be satisfied

$$\lim_{t \to \infty} E[\exp(-\beta t)J(k)] = 0. \tag{14}$$

It can be shown that the transversality condition is met for all solutions with positive consumption (see Merton, 1969).

We assume constant relative risk aversion of the individuals and time-invariance of all parameters of the model. Merton (1971) demonstrates that in this setting, capital and consumption grow at a common rate. Hence, the propensity to consume out of capital, μ, will be constant in macroeconomic equilibrium

$$c(t) = \mu k(t). \tag{15}$$

Now we can derive a closed-form solution for optimal consumption and determine expected growth, φ^*, from market clearing (7). To summarize the results of the planner's optimization, the following components of the macroeconomic equilibrium are derived:

$$\mu^* = \frac{\beta}{\rho} + \frac{\rho - 1}{\rho}\alpha^* + (\rho - 1)\alpha^{*2}\sigma^2\left(\eta^* - \frac{1}{2}\right), \tag{16}$$

$$\varphi^* \equiv \frac{E[dk]}{kdt} = \frac{1}{\rho}(\alpha^*(1 - \eta^*) \ - \beta) + \alpha^{*2}\sigma^2\left(\eta^* - \frac{1-\rho}{2}\right). \tag{17}$$

Since the optimality condition (12) immediately leads to a constant ratio $(g/k)^*$ in individual production, the optimal production elasticity of governmental input, η^*, as well as optimal productivity, α^*, is constant as well. This outcome confirms the conjecture of a time-invariant propensity to consume out of capital, which only depends on the underlying parameters as well as on the fiscal instruments.

With these results, capital is log-normally distributed

$$k(t) \sim \Lambda\left(\ln(k_0) + \left(\varphi^* - \frac{1}{2}\sigma_k^2\right)t, \sigma_k^2 t\right) \tag{18}$$

and evolves according to

$$k(t) = k_0 \exp\left(\left(\varphi^* - \frac{1}{2}\sigma_k^2\right)t + \sigma_k[z(t) - z(0)]\right). \tag{19}$$

As already shown above, the optimal ratios between consumption and capital, μ^*, and public input and capital, $(g/k)^*$, are constant. Hence, all relevant macroeconomic variables evolve in the same manner as determined in Equation (19).

The social optimum described by Equations (12), (16) and (17) can be attained in a competitive economy with an optimal policy mix. In what follows, the first-best optimum will be the reference for the evaluation of alternative tax policies.

IV. Market Equilibrium

We now turn to the optimization problem of a representative individual who maximizes expected lifetime utility (1). It is confronted by the production function (2), the congestion function (3) and the development of governmental bonds (4). With these assumptions there is an additional portfolio decision: The households have to decide which parts of individual wealth (as the sum of the two assets), w, they want to invest in physical capital and in government bonds. We assume that wealth is entirely distributed into the two assets. Thus, with the portfolio share of physical capital denoted by n, the portfolio share of government bonds is $1 - n$. The individual has to pay taxes on deterministic and stochastic income components, τ and τ', as well as a linear tax on consumption, ϖ.

The optimization problem of the individual then turns out to be

$$\max_{c,n,w} E_0\left[\int_0^{\infty} \frac{c^{1-\rho}}{1-\rho}\exp(-\beta t)dt\right] \tag{20}$$

$$\text{s.t.}\quad dw = [(1-\tau)\alpha n w + i(1-n)w - (1+\varpi)c]dt + (1-\tau')\alpha n w \sigma dz + (1-n)w dz_b. \tag{21}$$

The stochastic Bellman equation is given by

$$B = \exp(-\beta t)\frac{c^{1-\rho}}{1-\rho} - \beta\exp(-\beta t)J(w) + \exp(-\beta t)J'(w)[(1-\tau)\alpha n w + i(1-n)w - (1+\varpi)c] + \frac{1}{2}\exp(-\beta t)J''(w)\sigma_w^2, \tag{22}$$

with σ_w denoting the standard deviation of wealth. Intertemporal utility is maximized with respect to consumption, wealth and the portfolio share of physical capital, leading to the necessary conditions

$$c^{-\rho} = (1+\varpi)J'(w), \tag{23}$$

$$J'(w)((1-\tau)\alpha n(1-\varepsilon\eta) + (1-n)i - \beta) + \frac{1}{2}J'''(w)\sigma_w^2 + J''(w)\left((1-\tau)\alpha n w + i(1-n)w - (1+\varpi)c + \frac{\partial\sigma_w^2}{\partial w}\right) = 0, \tag{24}$$

$$J'(w)[(1-\tau)\alpha(1-\varepsilon\eta)w - iw] + J''(w)\frac{\partial\sigma_w^2}{\partial n} = 0. \tag{25}$$

Optimal consumption (23) together with the conjecture of a constant propensity to consume out of capital (15) lead to specific functions for the derivatives of the value function. As usual within this type of models, we suppose that in steady state the portfolio shares are constant. All assets grow at the same expected rate and furthermore face the same evolution of the stochastic processes. Hence, the equilibrium stochastic process of the real return of government bonds is

$$dz_b = \frac{\alpha\sigma}{1-n}(1-(1-\tau')n)dz. \tag{26}$$

Together with the arbitrage condition (25) for optimal portfolio choice, this implies the steady-state value of expected real return on bonds[3]

$$i = (1-\tau)\alpha(1-\varepsilon\eta) - \frac{\rho\alpha^2\sigma^2}{1-n}((1-\tau')(1-(1-n)\varepsilon\eta) - 1). \tag{27}$$

Implementing this relation in the necessary condition (24) leads to the following equation that describes the ratio μ between consumption and capital

$$(1+\varpi)\mu n = \frac{\beta}{\rho} + \frac{\rho-1}{\rho}(1-\tau)\alpha(1-\varepsilon\eta) + (1-\tau)\alpha\varepsilon\eta n \\ + \alpha^2\sigma^2\left((1-\tau')(1-\varepsilon\eta-\rho(1-(1-n)\varepsilon\eta)) + \frac{\rho-1}{2}\right). \quad (28)$$

It is constant in steady-state since both the production elasticity, η, and the portfolio choice are constant.[4] Now it is possible to determine the expected growth out of Equation (21)

$$\varphi \equiv \frac{E[dw]}{wdt} = \frac{1}{\rho}((1-\tau)\alpha(1-\varepsilon\eta)-\beta) + \alpha^2\sigma^2\left(\frac{\rho+1}{2} - (1-\tau')(1-\varepsilon\eta)\right). \quad (29)$$

The equilibrium real value of government debt, $(1-n)w$, is determined by the portfolio choice which defines the demand for government bonds. The portfolio share of physical capital is derived from the propensity to consume out of wealth (28) together with the ratio between consumption and capital from the market clearing condition (7). For an arbitrary supply of the public input, it is given by[5]

$$n = \frac{\beta - (1-\rho)(\varphi - \frac{1}{2}\rho\alpha^2\sigma^2)}{(1+\varpi)(\alpha - N^{-\varepsilon}(g/k) - \varphi) - \alpha\varepsilon\eta(1-\tau-\rho\alpha\sigma^2(1-\tau'))}. \quad (30)$$

All fiscal parameters (consumption tax, ω, as well as the income taxes, τ and τ') influence the portfolio decision and hence have an impact on the value of government debt. But since in equilibrium expected growth is independent of the portfolio choice, the value of government bonds is determined residually and does not affect intertemporal utility.

The important feature of government debt with respect to optimal fiscal policy is the indirect impact of income taxation on growth. The assumption of government bonds allows for stochastic revenues out of income taxation together with deterministic expenditures for the public input. In a closed economy the government cannot provide insurance against an aggregate productivity shock. A rise in the tax rate on stochastic income parts reduces the volatility of capital return but the government cannot eliminate the uncertainty. Instead, the volatility is shifted towards the return on government bonds. The opposite applies for tax cuts with respect to τ'.

In models that assume deterministic government expenditures together with stochastic tax revenues but neglect government debt, the stochastic part of the tax revenues is usually assumed to vanish.[6] That is, income taxation in this setting provides an insurance against production risk. Therefore, the response of risk averse agents to the taxation of stochastic income parts in those models is ambiguous and depends on relative risk aversion (see, for example, Smith (1996) for the learning-by-doing technology Ott and Soretz (2002) for the setting with a congested public input considered here, and

Soretz (2004) for a setting with stochastic pollution). As the tax rate on stochastic income components is an argument of the variance of wealth, expected growth turns out to be quadratic in this tax rate. If, instead, the governmental budget is closed through government debt, income taxation no longer affects the volatility of wealth. Hence, expected growth is linear in both income tax rates. There is no interior maximum of the growth rate with respect to the income tax rates.

V. Growth Effects of Tax Cuts

We are now able to analyze the growth effects of tax cuts. In order to assess this policy we derive the optimal levels of the taxes in the next section. Starting from the market equilibrium growth rate given in Equation (29), changes in the tax rates have the following growth effects:

$$\frac{\partial \varphi}{\partial \tau} = -\frac{\alpha}{\rho}(1 - \varepsilon\eta) < 0, \tag{31}$$

$$\frac{\partial \varphi}{\partial \tau'} = \alpha^2\sigma^2(1 - \varepsilon\eta) > 0, \tag{32}$$

$$\frac{\partial \varphi}{\partial \varpi} = 0. \tag{33}$$

The well-known result that taxing consumption is growth neutral in settings with inelastic labor supply is not changed by the introduction of risk and rivalry in the production function. Growth effects of tax cuts only arise when the tax base is individual income.

A reduction in the tax rate on deterministic income components leads to the growth enhancing effect, which is also known from deterministic growth models. This outcome is independent of the assumption about the government budget constraint and reflects the increase in net capital return. The level of congestion does not influence the sign of the growth effects in either case. A rise in congestion only increases the extent to which the growth effects of changes in income taxation arise. Our model does not replicate the inverted U shaped relationship between the tax rate and the growth rate, which is known from Barro (1990). This change in the results is due to the additional, growth neutral financing instruments in our model. Due to consumption tax and government debt, the level of the public input is independent from the income tax rate. An income tax cut does not necessarily decrease government expenditures and therefore, after-tax capital return remains unchanged.

With respect to the tax on stochastic income components, a tax cut increases the volatility of the net return on capital while the expected return on capital remains constant. Thus, taxation of stochastic capital returns induces a mean preserving spread as defined by Rothschild and Stiglitz (1970). The risk associated with the after tax return on capital increases and capital accumulation becomes less attractive for a risk averse agent. This reverses the insurance argument of taxation as first discussed by Domar and Musgrave (1944) and further developed by Stiglitz (1969). This kind of tax cuts induces a rise in the variance of returns. Therefore, it may reduce the demand for risky assets. Hence, with the assumption of government bonds which balance the government budget constraint, the growth effect of a tax cut on stochastic income components is unambiguously negative.[7] Formally this result may be derived in the case of uniform income taxation, $\tau = \tau'$, by the following relation

$$\left|\frac{\partial \varphi}{\partial \tau}\right| \gtrless \left|\frac{\partial \varphi}{\partial \tau'}\right| \Leftrightarrow 1 - \rho\alpha\sigma^2 \gtrless 0. \tag{34}$$

Hence the sign of the certainty equivalent determines the net growth effect of changes in the income tax rates. The certainty equivalent is assumed to be positive[8] and the growth effect of cuts in the income tax rates is unequivocally positive. Again, the level of congestion does not affect this result.

VI. Welfare Effects of Tax Cuts

For a welfare maximizing policy, it is not the level of the growth rate but the level of the fiscal instruments that gains importance. Tax cuts enhance welfare whenever the actual level of the tax rate is suboptimally high. If initially the fiscal policy is set optimally, tax cuts reduce welfare although they may enhance growth.

A welfare maximizing policy implies that decentral and optimal growth rates coincide. The governmental revenue must be sufficient to finance the optimal amount of the governmental input as determined by Equation (13). As shown in Equations (31)–(33) only the income tax rates induce changes in the growth rate. Consequently, τ and τ' are determined to realize the optimal level of the growth rate. The level of the consumption tax rate is then determined to close the governmental budget. Equalizing the growth rates in Equations (17) and (29) leads to the optimal relationship between the optimal tax rates on deterministic and stochastic income components

$$\tau^* = \rho\alpha^*\sigma^2\tau'^* + \frac{\eta^*(1-\varepsilon)}{1-\varepsilon\eta^*}(1-\rho\alpha^*\sigma^2). \tag{35}$$

It turns out to be linear. Condition (35) represents a continuum of optimal tax policies and the relation between the optimal tax rates is unambiguously positive. This reflects the contrarily interacting growth effects of the taxation of deterministic and stochastic income components. Starting from an optimal tax policy, a rise in the tax rate on deterministic income leads to a decline in expected growth, which can only be compensated by the positive growth effect of an increase in the tax rate on stochastic income.

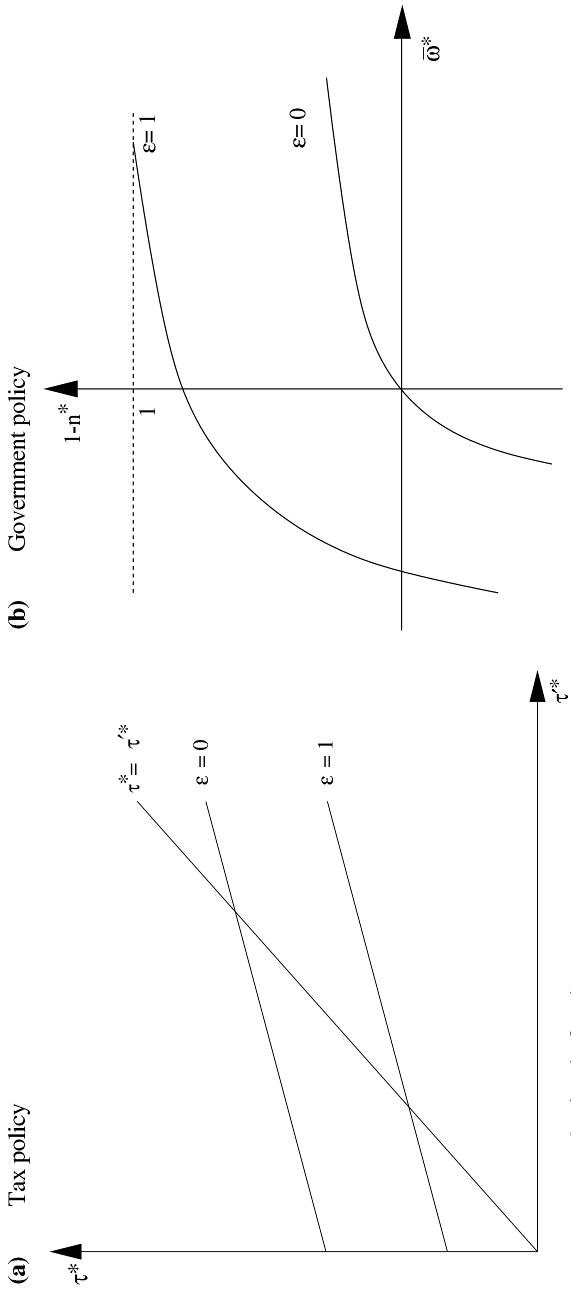

Figure 1. Optimal fiscal

For all optimal policies, the optimal value of government debt, $(1-n^*)w$, is determined by the equilibrium portfolio share of physical capital

$$n^* = \frac{\beta - (1-\rho)(\varphi^* - \frac{1}{2}\rho\alpha^{*2}\sigma^2)}{(1+\omega)(\beta - (1-\rho)(\varphi^* - \frac{1}{2}\rho\alpha^{*2}\sigma^2)) + \alpha^*\eta^*(1-\rho\alpha^*\sigma^2)\frac{\varepsilon(1-\eta^*)}{1-\varepsilon\eta^*}}. \tag{36}$$

It depends on the consumption tax rate, uncertainty and rivalry. The production elasticity, η^*, is evaluated at the optimal level of government expenditure as given by Equation (12). The welfare maximizing expected growth rate, φ^*, corresponds to Equation (17).

We now analyze how uncertainty and rivalry influence optimal fiscal policy. The optimal tax policy (35) is demonstrated in Figure 1(a) for alternative levels of congestion. The two parallel lines show the special cases of a pure public and a pure private good. The remaining cases ($0 < \varepsilon < 1$) are found in between. For feasible solutions the slope is less than unity as can be seen from Equation (12). The third line, $\tau^* = \tau'^*$ reflects the case of uniform taxation of deterministic and stochastic income parts.

If there is no congestion ($\varepsilon = 1$), the negative externality of capital accumulation as given in the congestion function (3) vanishes. The individuals realize optimal expected growth in market equilibrium, for example, for an optimally determined level of the expenditure ratio $G/(N\bar{y})$ that is exclusively financed by taxing consumption. Hence, optimal uniform taxation of deterministic and stochastic income requires the absence of any income taxation, $\tau^* = \tau'^* = 0$. This outcome reflects the result in the deterministic setting where optimal fiscal policy in the case of a pure public good leads to complete financing via a growth neutral consumption tax. With a differential tax rates, $\tau \neq \tau'$, there is a continuum of optimal tax policies. All fiscal policies that meet Equation (35) are equivalent with respect to expected intertemporal utility. The higher the taxation of average income, the higher the taxation of random income must be. Condition (35) ensures that the distortionary impact of a positive tax rate on deterministic income components is offset by the growth enhancing insurance effect of a positive tax rate on stochastic income parts.

If, instead, congestion is proportional ($\varepsilon = 0$), the equilibrium growth rate is suboptimally high due to the negative external effect of capital accumulation. The representative household neglects his influence on aggregate capital and overestimates the individually available amount of the public input. Hence, the expected private rate of return on capital as well as the volatility of private capital return are higher than their social counterparts. This incidence gives rise to a tax policy that reduces the net marginal return on capital. Using condition (12) as well as the relation $g = G/N$ leads to

$$\varepsilon = 0 \;\Rightarrow\; \tau^* = \rho\alpha^*\sigma^2\tau'^* + \frac{G}{N\bar{y}}. \tag{37}$$

A policy without taxation of stochastic income parts ($\tau'^{*} = 0$) is optimal if government expenditure is fully financed by an exclusive taxation of deterministic income. It is thus possible to replicate the result of the deterministic model. If, instead, the tax rate on stochastic income is positive, the optimal tax rate on deterministic income increases. The reason for this given fact is that taxation of uncertain income components leads to an increase in expected growth, thus driving it away from the Pareto-optimal growth rate. Hence, the tax rate on deterministic income parts has to be higher in order to compensate for this additional positive growth effect.

For alternative income tax policies, the corresponding portfolio share of government bonds, 1-n^{*}, depends on the level of the consumption tax rate, ϖ. It is illustrated in Figure 1(b). Again, the two polar cases of proportional congestion and absence of any rivalry are shown as benchmarks. All other cases of partial congestion can be found to lie between these two lines. It becomes obvious that an increase in congestion leads to a decrease in the real value of government debt for all levels of ϖ^{*}. Since, with rising congestion, the negative externality of capital accumulation increases, internalization via income taxation becomes more important to realize the first-best optimum. Revenues out of income taxation increase. Ceteris paribus this leads immediately to a reduction in government debt. Then, with a rise in the consumption tax, ϖ^{*}, government revenues increase even more. Since for any optimal fiscal policy, government expenditures remain constant at the optimal level, real interest payments on government bonds must rise. That is, the real value of government bonds increases for all levels of the consumption tax. In this respect, our model includes the results derived by Turnovsky (1999): Without government debt, the consumption tax is used to close the governmental budget. Hence, a cut in one tax rate can be balanced by an increase in another tax rate. The special case without government debt is depicted in Figure 1(b) along the abscissa. A reduction in rivalry in this case goes along with a compulsory increase in the optimal level of the consumption tax rate.

Note again the case where a flat rate income tax is levied, $\tau^{*} = \tau'^{*}$, as illustrated in Figure 1(a). In order to internalize the external effect, the optimal income tax rate is positive. Equation (37) shows that the optimal flat rate increases with uncertainty. The optimal tax rate determined under certainty underestimates the need for growth reducing income taxation. In particular, the expected revenue from the income tax is higher than the optimal expenditures for the public input. The remaining part is redistributed in a growth neutral way, for example through a subsidy on consumption.

If congestion is neither absent nor complete ($0 < \varepsilon < 1$), the line of optimal tax policy is situated between the two lines in Figure 1(a) and government debt is located between the two graphs in Figure 1(b). Again, there is a continuum of optimal fiscal policies with a positive relation between the

Table I. Optimal fiscal policies

	Proportional rivalry (ε = 0)	No rivalry (ε = 1)
τ^*	$\rho\alpha^*\sigma^2\tau'^* + \frac{G}{N\bar{y}}$	$\rho\alpha^*\sigma^2\tau'^*$
τ'^*	$\frac{\tau^* - G/(N\bar{y})}{\rho\alpha^*\sigma^2}$	$\frac{\tau^*}{\rho\alpha^*\sigma^2}$
ϖ^*	$\frac{1-n^*}{n^*}$	$\frac{1-n^*}{n^*} - \frac{\alpha^*\eta^*(1-\rho\alpha^*\sigma^2)}{\beta-(1-\rho)(\varphi^*-\rho\alpha^*\sigma^2/2)}$

two tax rates. The suboptimally high equilibrium growth rate is reduced via taxation of deterministic income parts. In the case of a positive tax rate on stochastic income, the growth enhancing insurance effect has to be compensated for additionally by a higher tax rate on expected income. Hence, an optimal flat rate income tax will be increased by uncertainty in any case of partial congestion.

Tax cuts applied to the income revenues should only be used if the actual levels of the income taxes exceed their optimal levels. If the government is not able to distinguish deterministic and random income parts, it will choose uniform income tax rates, $\tau = \tau'$. Their optimal levels increase with a rise in congestion. If the public input is subject to congestion ($\varepsilon < 1$), the positive income taxes internalize the negative effect of capital accumulation. Tax cuts should not reduce the income taxes under the given positive level τ^*. If no congestion arises ($\varepsilon = 1$) governmental tax cuts should reduce the income tax rate and fully finance governmental expenditure via the tax on consumption. At the same time the government must guarantee that the amount of the public input does not become suboptimally small. Then, the income tax is the right base for tax cuts as with an increase in the growth rate welfare is also enhanced.

The results for the benchmark cases of proportional congestion and no congestion are summarized in Table I. To calculate the optimal level of the consumption tax rate, Equation (36) has been used. This also points to the effects of public debt financing.

VII. Summary

We show that income tax cuts induce ambiguous consequences for equilibrium growth in a stochastic dynamic economy with a congested governmental input. A reduction in the tax on deterministic income components leads to the politically desirable positive growth effect. But in contrast, a decrease in the taxation of stochastic income parts induces a decline in equilibrium capital accumulation. We develop the social optimum and contrast the outcomes with the market equilibrium. Afterwards, we analyze the growth effects of tax cuts and develop conditions for optimal fiscal policies.

This allows for a welfare analysis of certain tax policies. Due to congestion there is a negative externality of capital accumulation. The representative individual neglects his influence on aggregate capital and hence a part of his impact on the availability of the public input. Therefore, marginal return on capital is overestimated. Uncertainty has two major impacts. First, any rise in the governmental input does not only increase the expected value of capital productivity but also the volatility of marginal capital return. For this reason the optimal ratio between governmental input and physical capital is reduced by uncertainty. Second, any increase in income tax influences the volatility of after tax capital income and with this also the optimal reaction of a risk averse individual.

Since we assume a differential income tax rate there is a continuum of optimal tax policies. According to the degree of rivalry, any optimal fiscal policy sets the income tax rates in order to adjust equilibrium growth to optimal growth. Income taxation is used to internalize the external effect of capital accumulation. Hence, the part of the public input that should be financed via income taxation from the point of view of welfare economics depends on the extent of the negative externality. It is also possible to determine those revenues that should be the base for tax cuts in order to achieve an optimal policy. The growth effects then do not play a major role.

The endogenous value of optimal government debt is derived. It is shown to depend positively on the consumption tax rate and to decrease with a rise in rivalry. Additionally, we consider the special case of a flat rate income tax. The optimal flat rate is zero if there is no congestion and increases with a rise in the degree of rivalry. Due to the positive growth effect of the tax on stochastic income parts, the optimal flat rate exceeds expected expenditure for the public input in case of proportional congestion. Optimal policy in this case includes a consumption subsidy and then consumption and not income should be the base for tax cuts in order to allow for the optimal amount of the public input.

Notes

1. For an analysis of stochastic growth in the case of non-expected utility, where relative risk aversion can deviate from the reciprocal of the intertemporal elasticity of substitution, see, for example, Obstfeld (1994), Smith (1996) or Clemens and Soretz (1999).
2. The certainty equivalent is $\alpha(1-\eta)(1-\rho\alpha\sigma^2)$ for the planner solution. For the determination see, for example, Merton (1992, p. 45).
3. This outcome only applies as long as $n \neq 1$. In the case of $n = 1$, the value of government bonds is zero, $b = 0$. This implies no taxation of stochastic capital returns, $\tau' = 0$, since otherwise the government budget could not be balanced. The stochastic process of return on bonds then vanishes, too ($dz_b = 0$). With this setting, the expected real return on bonds is $i = (1-\tau)\alpha(1-\varepsilon\eta) - \rho\alpha^2\sigma^2(1-\varepsilon\eta)$.
4. The production elasticity, η, is constant in equilibrium even if governmental activity is set arbitrarily. Note that in equilibrium all relevant economic variables grow with the same

stochastic rate. Moreover, all N individuals are assumed to be homogeneous. That is, aggregate capital K is represented by Nk. Hence, the ratio g/k remains constant in any equilibrium, independent of the level of governmental activities.

5. The denominator is positive for feasible solutions because it is equivalent to the transversality condition. Furthermore, a well-defined equilibrium with a positive capital stock requires a positive portfolio share of capital. Hence, we consider only policy parameters that ensure a positive denominator of n.
6. This assumption turns out to be much more questionable in stochastic models than in deterministic ones as the destruction of stochastic tax revenues with zero expected value implies the creation of revenues from nowhere in all time increments with negative realization of the productivity shock.
7. This outcome changes substantially with the neglect of government debt as discussed in Ott and Soretz (2002), for example. If there are no governmental bonds to close the budget the growth effect of changes in the tax on random income parts is ambiguous and depends on the degree of risk aversion, see also Turnovsky (1999). Within this paper, the level of congestion does not influence the sign of changes in τ' but the extent to which the growth rate changes.
8. See footnote 2 where the certainty equivalent of the planner solution is determined.

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