The Manchester School Vol 71 No. 4 Special Issue 2003
1463–6786 448–469

STOCHASTIC POLLUTION AND ENVIRONMENTAL CARE IN AN ENDOGENOUS GROWTH MODEL

by
*SUSANNE SORETZ**
University of Hannover

The impact of pollution and abatement policy within a stochastic endogenous growth model is analyzed. Environmental care is provided by the government and financed through income taxation and government bonds. Due to environmental preferences and partial perception of the individual's impact on pollution, government debt influences equilibrium growth. Hence, there is an additional growth effect of income taxation due to portfolio adjustment. It is shown that the optimal income tax rate decreases with the perception of the influence of individuals on aggregate capital. In contrast, the impact of environmental preferences and uncertainty on optimal environmental policy is ambiguous.

1 INTRODUCTION

Recent contributions analyze the interdependence between environmental aspects and endogenously determined growth. Depending on the specific assumptions, pollution tends to decrease optimal growth. Eventually, growth may even cease due to increasing environmental costs of production (e.g. Stokey, 1998). On the other hand, pollution can lead to an increase in optimal growth, since more growth enhances the possibilities for future abatement activities (e.g. Smulders and Gradus, 1996).

The Ramsey problem of pollution in a dynamic economy is based on Forster (1973), whose framework was extended by for example Gruver (1976), Luptacik and Schubert (1982), Siebert (1987) and Van der Ploeg and Withagen (1991). These authors analyze the effects of pollution in neoclassical growth models. The common outcome is that pollution induces a decline in the optimal steady-state capital stock. Nevertheless, the question whether or not environmental concerns are consistent with ongoing growth can only be addressed within the setting of endogenous growth. Gradus and Smulders (1993) as well as Stokey (1998) consider environmental preferences and various technologies with constant returns in the accumulable inputs to allow for ongoing growth. Gradus and Smulders (1993) show a negative relation between optimal growth and pollution disutility in the case of constant returns to capital and with endogenous abatement activities which determine

---

*I am indebted to two anonymous referees for helpful comments and suggestions.

©Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.
Published by Blackwell Publishing Ltd, 9600 Garsington Road, Oxford OX4 2DQ, UK, and 350 Main Street, Malden, MA 02148, USA.

the level of pollution. In contrast, they demonstrate that environmental preferences in the framework of Lucas (1988) have no effect on long-run growth. If additionally learning ability is reduced by pollution, abatement can even enhance growth in the case where the rise in capital productivity dominates the negative growth effect associated with the crowding out of investment. Stokey (1998) analyzes a model with linear production technology and endogenous emission standards. Pollution is assumed to be proportional to total output. In this setting, sustained growth is not optimal, i.e. growth ceases in the long run. During transition, pollution is an asymmetric hump-shaped function of income, as empirically found by Grossman and Krueger (1995).

The paper presented here introduces two important features to an endogenous growth model with pollution (which is based on Gradus and Smulders, 1993). First, uncertainty is incorporated in the model, because it is an essential quality of environmental degradation. It is shown that the outcomes of the corresponding deterministic model cannot be applied to the case of uncertainty, except for logarithmic preferences. Second, the perception of the individual's influence on aggregate pollution is parameterized. This assumption reflects a partial individual sense of responsibility for environmental degradation and will be explained in more detail below.

Uncertainty about future consequences of present actions is of particular interest in the context of environmental issues. The impact of uncertainty on the growth process was first analyzed by Levhari and Srinivasan (1969), Brock and Mirman (1972, 1973) and Leland (1974) in a Ramsey-type economy with utility-maximizing individuals. The authors predominantly focused on conditions for stability of the growth process. More recently, economic growth under uncertainty was reconsidered by for example Greenwood and Jovanovic (1990), Devereux and Smith (1994), Obstfeld (1994), Clemens and Soretz (1999), de Hek (1999) and Jones *et al.* (1999) within the endogenous growth setting. It was shown that the growth effect of uncertainty is ambiguous and depends on the parameter setting—predominantly on the degree of relative risk aversion. This result continues to hold in the model presented here and additionally leads to an ambiguous influence of uncertainty on optimal abatement activity.

Furthermore, the impact of any environmental policy depends crucially on the existence of uncertainty. Governmental activities influence not only the expected values of economic variables but also their volatility and in general this leads to counteracting effects on market equilibrium. These were first discussed by Eaton (1981) who analyzes the ambiguous impact of income taxation in a stochastic growth model with linear technology. More recent papers, e.g. Turnovsky (1993, 1995a, 1999, 2000), Smith (1996), Corsetti (1997) and Clemens and Soretz (1997), extend this framework and demonstrate the role of governmental activities in various settings of stochastic endogenous growth.

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

There are few contributions which focus on the dynamic consequences of uncertainty on pollution and abatement. Clarke and Reed (1994) discuss the implications of an environmental catastrophe, whereas Baranzini and Bourguignon (1995) analyze the probability of survival which is influenced by environmental degradation. Uncertainty about future preferences is analyzed by Beltratti *et al.* (1998) as well as Ayong Le Kama (2001). In contrast, Chichilnisky and Heal (1998) focus on unknown risks and Reis (2001) discusses the positive probability of discovering a clean technology. In the setting analyzed here, uncertainty is driven by a productivity shock. The level of future pollution is uncertain, since output as well as capital stock evolve stochastically. This assumption reflects the fact that the link between output and pollution is not stationary.

The second important feature incorporated in the model refers to the perception of individual influence on aggregate pollution. Emissions reduce individual expected utility and hence social welfare. The level of pollution depends on the relation between abatement activities and the capital stock. The extent to which the agent perceives pollution to depend on individual capital is parameterized and both polar cases are included: on the one hand the case where agents are aware of the true relation between pollution and individual capital accumulation and on the other hand the case where pollution is perceived as purely exogenous to individual decisions. This setting of perception relates to the formulation of congestion functions in the public goods literature, as for example in Edwards (1990), Glomm and Ravikumar (1994) and Turnovsky and Fisher (1998).

Since there is a continuum of identical individuals and abatement is non-rival and non-excludable, the representative agent neglects his/her contribution to aggregate abatement. Thus, environmental care is assigned to government and financed via income taxation and government bonds. Due to uncertainty, the revenues out of income taxation are volatile. As long as abatement policy is assumed to be deterministic, i.e. environmental policy shocks are excluded, the governmental budget cannot be balanced in each time increment but, instead, has to be closed by government debt.

Building on the assumptions of the model described in Section 2, the market equilibrium is analyzed in Section 3. Due to the environmental preferences capital accumulation has a secondary, welfare-diminishing effect which in part—depending on the perception of capital in the pollution function—is external. Hence, in contrast to most stochastic endogenous growth models, expected growth depends on equilibrium portfolio composition and thus on government debt. The usual separability of the growth process from portfolio choice, which is due to Ricardian equivalence and reflects the fact that the value of government bonds is determined by arbitrage arguments, does not hold in this model. Instead this interdependence leads to various interacting effects of fiscal policy on macroeconomic equilibrium.

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

Section 4 analyzes the growth effects of fiscal policy. It can be concluded that the optimal share of tax financing increases with the perception of aggregate capital as exogenous and reacts in an ambiguous way with environmental preferences and risk aversion respectively. Section 5 discusses the polar case where individuals ignore completely the influence of private capital accumulation on pollution. This setting is emphasized because it corresponds to the usual assumption of deterministic endogenous growth models with pollution and particularly to Gradus and Smulders (1993). Finally, Section 6 concludes.

2 THE MODEL

Pollution causes disutility and hence damages social welfare. With this formulation, the paper builds on the analysis of Gradus and Smulders (1993) as well as Stokey (1998), and draws back on the earlier approaches of Luptacik and Schubert (1982) and Van der Ploeg and Withagen (1991). Consider a continuum of identical infinitely long-lived households who maximize expected lifetime utility

$$U = E_0\left\{\int_0^\infty \exp(-\rho t)u[c(t), P(t)]\mathrm{d}t\right\} \tag{1}$$

One could think of a long-lived dynasty, because altruism between generations is very plausible in the context of environmental issues. Consumption is represented by $c(t)$ and pollution by $P(t)$. The rate of time preference $\rho > 0$ is assumed to be constant. E_0 denotes the expected value conditional on time 0 information.

Instantaneous utility exhibits constant relative risk aversion:

$$u[c(t), P(t)] = \begin{cases} \dfrac{\left[c(t)P(t)^{-\gamma}\right]^{1-\varepsilon}}{1-\varepsilon} & \text{for } \varepsilon \neq 1 \\ \ln c(t) - \gamma \ln P(t) & \text{for } \varepsilon = 1 \end{cases} \tag{2}$$

Environmental preferences come into play for $\gamma > 0$, and with higher γ disutility out of pollution gains importance. $\varepsilon > 0$ represents the degree of relative risk aversion.

The homogeneous good is produced according to the linear individual stochastic production function

$$f[k(t)] = Ak(t)\left[\mathrm{d}t + \sigma_y \mathrm{d}z(t)\right] \tag{3}$$

The deterministic counterpart of this technology was popularized by Rebelo (1991) and the stochastic version draws back to Eaton (1981). Uncertainty enters the economy via an aggregate productivity shock $\mathrm{d}z(t) \sim \mathrm{N}(0, \mathrm{d}t)$, which is the increment to a Wiener process. A is expected capital productivity and $k(t)$ denotes a broad measure of capital available to the representative firm.

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

Pollution depends on the relation between capital and abatement activities, and it is modeled as a flow variable. The formulation is based on Gradus and Smulders (1993). In this paper, the approach is extended and the perception of the capital stock is taken into account. Pollution $P(t)$ is determined by the relation of aggregate capital $K(t)$, individual capital $k(t)$ and abatement effort $E(t)$:

$$P(t)=\left[\frac{K(t)^{\delta}k(t)^{1-\delta}}{E(t)}\right]^{\alpha}\mathrm{d}t \qquad \alpha>0, \qquad \delta\in[0,1] \tag{4}$$

The extent to which the agent perceives pollution to depend on exogenous aggregate capital is parameterized by δ. In equilibrium, aggregate capital equals individual capital, because all households are identical and the population size is normalized to unity. Thus, $\delta = 0$ represents perfect knowledge about the effect of individual capital accumulation on pollution. On the other hand, $\delta = 1$ is associated with the situation where the representative agent completely neglects his/her individual contribution to aggregate capital. Pollution is perceived not to depend in any way on individual accumulation. For $0 < \delta < 1$ perception of capital is in between these polar cases. Individual capital is expected to influence pollution, but the extent of the dependence is underestimated.

Pollution control, $E(t)$, is assumed to be non-rival and non-excludable. Thus, with a continuum of households, the effect of individual effort on environmental quality vanishes. Hence, abatement activity is provided by the government and financed by income taxation and government debt. Furthermore—in order to keep the model simple—abatement activity is assumed to be instantaneously deterministic, i.e. there is no environmental policy shock. The only source of uncertainty is the productivity shock. In accordance with the usual assumptions on all fiscal instruments, the households suppose abatement expenditures to be exogenous to their decisions. Consequently, the market equilibrium of the next section is determined for an arbitrary amount of abatement expenditures, while the discussion of optimal policies is deferred to Section 4.

The government levies a proportional income tax at the rate τ. With stochastic production, tax revenues out of income taxation are stochastic whereas expenditures for abatement activities are deterministic. Hence, the government budget cannot be balanced at each instant of time. Instead, it is closed by issuing bonds with stochastic value $B(t)$. These bonds are assumed to be perpetuities which pay an after-tax expected interest rate i as well as a net stochastic interest rate $\mathrm{d}z_i$. Both parts of the return on government bonds have to be determined in equilibrium. The value of government bonds evolves according to

$$\mathrm{d}B(t)=[E(t)+iB(t)-\tau AK(t)]\mathrm{d}t+B(t)\mathrm{d}z_i-\tau AK(t)\sigma_y\mathrm{d}z \tag{5}$$

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

In the following section, individual optimization and the resulting macroeconomic equilibrium will be analyzed.

3 MARKET EQUILIBRIUM

In the presence of government bonds, the representative individual not only has to choose optimal capital accumulation but also the utility-maximizing portfolio. Individual wealth $w(t)$ is composed of physical capital $k(t)$ and financial wealth $b(t)$. The portfolio share of capital is denoted by n. Thus, the individual wealth restriction is

$$\mathrm{d}w = [(1-\tau)Anw + i(1-n)w - c]\mathrm{d}t + (1-\tau)Anw\sigma_y \mathrm{d}z + (1-n)w\mathrm{d}z_i \tag{6}$$

and the variance of wealth (divided by $\mathrm{d}t$) is

$$\sigma_w^2 = (1-\tau)^2 A^2 n^2 w^2 \sigma_y^2 + 2(1-\tau)An(1-n)w^2\sigma_{yi} + (1-n)^2 w^2 \sigma_i^2 \tag{7}$$

where σ_{yi} denotes the covariance between output and return on bonds and σ_i^2 is the variance of the return on bonds. The individuals suppose these both to be exogenous to their decisions, although they depend on the endogenous stochastic process of the return on bonds, $\mathrm{d}z_i$, which has to be determined in equilibrium.

Expected intertemporal utility (1) is maximized with respect to the wealth constraint (6) while tax rates as well as initial values k_0 and z_0 are given. With the specification for utility given above, the maximized lifetime utility can be shown to be of the time-separable form $\exp(-\rho t)J[w(t)]$. Employing Itô's lemma, the stochastic Bellman equation is given by

$$\begin{aligned}\mathfrak{B} = \exp(-\rho t)u(c,P) - \rho\exp(-\rho t)J(w) + \exp(-\rho t)J'(w)\frac{E(\mathrm{d}w)}{\mathrm{d}t} \\ + \frac{1}{2}\exp(-\rho t)J''(w)\sigma_w^2\end{aligned} \tag{8}$$

Optimization for the representative individual requires the expected utility-maximizing choice of consumption and portfolio. There is no individual choice about the level of abatement activities, since the individual influence on environmental restoration is neglected. Hence, maximization leads to the first-order conditions with respect to consumption, c, and portfolio share of capital, n,

$$c^{-\varepsilon}P^{-\gamma(1-\varepsilon)} = J'(w) \tag{9}$$

$$\frac{\alpha\gamma(1-\delta)}{n}c^{1-\varepsilon}P^{-\gamma(1-\varepsilon)} + J'(w)[(1-\tau)Aw - iw] + \frac{1}{2}J''(w)\frac{\partial\sigma_w^2}{\partial n} = 0 \tag{10}$$

together with the transversality condition

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

$$\lim_{t\to\infty} E[\exp(-\rho t)J(W)] = 0 \tag{11}$$

which has to be satisfied for expected utility to be bounded and in order to ensure feasible consumption paths.

Equation (9) implies the equality of marginal utility out of consumption over time. For a setting which satisfies the conditions (i) time-invariant relative risk aversion, (ii) constant marginal product of capital and (iii) variance proportional to the square of the state variable, Malliaris and Brock (1982, p. 178) show that there exists a closed-form solution. In the setting considered here, these conditions are met, as can be seen from equations (2), (3) and (7). The value function then has the same shape as the instantaneous utility function. This outcome can be demonstrated by the conjecture of a constant relation between consumption and wealth, which is denoted by μ. Hence, substitution of $c = \mu w$ in condition (9) results in

$$J'(w) = (\mu w)^{-\varepsilon} P^{-\gamma(1-\varepsilon)} \tag{12}$$

The solution for market equilibrium again is by conjecture. Constant relative risk aversion together with constant expected rates of return and standard deviations of the returns which are proportional to the level of wealth lead to the guess of constant portfolio choice. In the following, the existence of an equilibrium of this type will be proved. If the portfolio shares are constant, all assets grow with the same stochastic rates:

$$\frac{\mathrm{d}w}{w} = \frac{\mathrm{d}k}{k} = \frac{\mathrm{d}b}{b} \tag{13}$$

That is, expected growth rates are equal and the stochastic processes evolve in the same way. One would expect transitional dynamics at this point of the analysis, but they can be ruled out with a simple further assumption about the bonds (see Turnovsky, 1995b, p. 403 together with p. 450). To enhance the tractability of the equilibrium, the government bonds are assumed to be perpetuities, i.e. the number of bonds is constant and an initial price jump at $t = 0$ adjusts the value of bonds b in order to satisfy $b_0 = [(1 - n)/n]k_0$. For all $t > 0$, the price of bonds (and thus their value) grows at the common equilibrium stochastic rate.

In particular, equating the stochastic components of (13) leads to the equilibrium stochastic process of bond returns

$$\mathrm{d}z_i = \frac{1-(1-\tau)n}{1-n} A\sigma_y \mathrm{d}z \tag{14}$$

which is proportional to the productivity shock. Hence, if an equilibrium with constant portfolio shares exists, the stochastic process of the value of bonds adjusts to the volatility of productivity according to (14).

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

With the stochastic process of bond returns given in equation (14) the variance of the return on bonds results immediately in $\sigma_i^2 = [1 - (1 - \tau)]^2 n^2 A^2 \sigma_y^2 / (1 - n)^2$ and the covariance between output and return on bonds is given by $\sigma_{yi} = [1 - (1 - \tau)n] A \sigma_y^2 / (1 - n)$. Substitution of equation (12) together with σ_i^2 and σ_{yi} into equation (10) now yields an arbitrage condition which determines a unique relation between the expected rate of return on bonds and portfolio choice:[1]

$$i = (1-\tau)A + \varepsilon \frac{A^2\sigma_y^2\tau}{1-n} - \alpha\gamma(1-\delta)\mu^{\varepsilon} \tag{15}$$

For optimal wealth accumulation, the derivative of (8) with respect to wealth has to be zero. Replacing the derivatives of the value function with conjecture (12) and the return on bonds with (15) leads to the consumption–wealth ratio

$$\mu = \frac{\rho + (\varepsilon-1)(1-\tau)A - \frac{1}{2}\varepsilon(\varepsilon-1)(1-2\tau)A^2\sigma_y^2}{\varepsilon - \alpha\gamma(1-\delta)[1-\varepsilon(1-n)]/n} \tag{16}$$

which is indeed constant and confirms the conjecture as long as portfolio composition is constant over time. Portfolio choice will be determined endogenously in equilibrium and will turn out to be constant (see equation (25)).

The propensity to consume out of wealth depends on the underlying parameters as well as on the fiscal instruments. Two special cases can be noted here. If aggregate capital is perceived to be completely exogenous to the accumulation decision of the individual, $\delta = 1$, or if the environmental preferences vanish, $\gamma = 0$, the second term in the denominator is zero and the consumption–wealth ratio corresponds to the linear model without pollution. In the first case the individuals are not aware of their influence on disutility out of pollution. In the second case there is no negative impact of pollution on utility.

The expected growth rate of the economy, φ, can be obtained from the individual wealth constraint (6)

$$\begin{aligned}\varphi \equiv \frac{E(\mathrm{d}w)}{w\mathrm{d}t} &= \frac{1}{\tilde{\varepsilon}}[(1-\varepsilon+\tilde{\varepsilon})(1-\tau)A - \rho] \\ &\quad + \frac{\varepsilon}{2\tilde{\varepsilon}}[\varepsilon - 1 + 2\tau(1-\varepsilon+\tilde{\varepsilon})]A^2\sigma_y^2\end{aligned} \tag{17}$$

---

[1] Note that equations (14) and (15) are only valid in the case $n \neq 1$. Otherwise, there is no government debt and therefore the government budget is balanced at each instant of time. This will only be a feasible equilibrium if the taxation of stochastic income parts vanishes, i.e. τ is only applied to expected income. Government bonds are then a sure asset ($\mathrm{d}z_i = 0$) and the resulting rate of return is $i = (1 - \tau)A - \varepsilon A^2\sigma_y^2$.

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

with $\tilde{\varepsilon}$ defined as follows:

$$\tilde{\varepsilon} \equiv \frac{\varepsilon[n+\alpha\gamma(1-\delta)(1-n)]-\alpha\gamma(1-\delta)}{n+\alpha\gamma(1-\delta)(1-n)} \leq \varepsilon \tag{18}$$

To ensure feasible solutions, $\tilde{\varepsilon}$ is assumed to be positive for all values of relative risk aversion ε. Therefore, parameters have to fulfill the condition $1 - \alpha\gamma(1 - \delta) > 0$ for a positive perceived marginal productivity of capital. Hence, environmental preferences should not be too strong.

The first term of the expected growth rate (17) equals the growth rate of the corresponding deterministic model. The second term reflects the response of the risk-averse representative individual to uncertainty in future income flows and displays the motive for precautionary savings (for a detailed discussion of the growth effects of uncertainty see, for example, Eaton, 1981; Obstfeld, 1994; Turnovsky, 1995a; Clemens and Soretz, 1999).

What is the Effect of Pollution on Equilibrium Growth? Due to pollution which is positively related to the capital stock, there is a negative external effect of capital accumulation.[2] This has an ambiguous effect on expected growth. Since the average capital return is overestimated, growth tends to be suboptimally high without taxation. In contrast, the overestimation of the risk associated with capital returns, *ceteris paribus*, leads to suboptimally low accumulation. As long as the certainty equivalent of the portfolio return is positive,[3] the first effect dominates and the negative externality of capital accumulation leads to suboptimally high growth.

How Do Private Perception and Environmental Preferences Influence Growth? The impacts of perception of aggregate capital as well as of environmental preferences are derived from the growth rate (17):

$$\frac{\partial\varphi}{\partial\delta} = \frac{n\alpha\gamma}{\tilde{\varepsilon}}\left[(1-\tau)A\left(1-\varepsilon A\sigma_y^2\right)-\varphi\right] > 0 \tag{19}$$

$$\frac{\partial\varphi}{\partial\gamma} = \frac{n(1-\delta)}{\tilde{\varepsilon}}\left[(1-\tau)A\left(1-\varepsilon A\sigma_y^2\right)-\varphi\right] < 0 \tag{20}$$

Both effects depend on the relation between expected growth and the certainty equivalent of the portfolio return. In analogy to deterministic endogenous growth models, the transversality condition implies that the certainty equivalent of the portfolio return has to exceed expected growth for feasible

---

[2] In the case of perfect anticipation, $\delta = 0$, this additional negative effect of capital accumulation is completely internalized within the individual intertemporal decision.

[3] The certainty equivalent of the portfolio return can be evaluated according to Merton (1992, p. 45) and is positive if $1 - \varepsilon A\sigma_y^2 > 0$.

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

solutions. Therefore, the growth rate increases with the perception parameter δ and decreases with the environmental preference parameter γ. With higher δ the individuals perceive less influence of individuals on aggregate capital. That is, they anticipate a smaller effect of capital accumulation on pollution and on disutility out of pollution. This leads to an increase in equilibrium accumulation.

With respect to environmental preferences, the impact is similar. Increasing importance of pollution for utility (i.e. a higher γ) increases the cost of capital accumulation. Since the negative effect of capital accumulation is given more weight in optimization, savings are diminished.[4]

The productivity shock is assumed to be a Wiener process. Hence, time t capital is a geometric Wiener process and is log-normally distributed. Given the initial values of capital k_0 and the stochastic process z_0 at time 0, capital evolves according to

$$k(t) = k_0 \exp\left\{\left(\varphi - \tfrac{1}{2}A^2\sigma_y^2\right)t + A\sigma_y[z(t) - z_0]\right\} \tag{21}$$

Maximal expected lifetime utility is then given by

$$U = \frac{(A - \varphi - \eta)^{1-\varepsilon}\eta^{\alpha\gamma(1-\varepsilon)}k_0^{1-\varepsilon}}{(1-\varepsilon)\left[\rho - (1-\varepsilon)\left(\varphi - \tfrac{1}{2}\varepsilon A^2\sigma_y^2\right)\right]} \tag{22}$$

where the relation between abatement and physical capital, which is set by the government and will be determined in the next section, is denoted by $\eta = E/k$ and market clearing requires $c/k = A - \varphi - \eta$.

4 FISCAL AND ENVIRONMENTAL POLICY

This section builds on the determination of the dynamic equilibrium and analyzes the impact of income taxation and government debt on steady-state growth. Afterwards, conditions for optimal environmental policy are stated.

4.1 Growth Effects of Taxation

Income taxation has ambiguous impacts on equilibrium expected growth. There is both a direct and an indirect growth effect of income taxation:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} = \underbrace{\frac{\partial\varphi}{\partial\tau}}_{\text{direct effect}} + \underbrace{\frac{\partial\varphi}{\partial n}\frac{\partial n}{\partial\tau}}_{\text{indirect effect}} \tag{23}$$

---

[4] Note that this effect is not as straightforward as it seems. As soon as abatement effort is determined endogenously, the growth effect may become ambiguous; see the explanation given with equation (30) below or by Smulders and Gradus (1996) for the deterministic setting.

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

The direct impact of a change in the income tax rate is the well-known ambiguous growth effect of income taxation in a stochastic growth model:

$$\frac{\partial \varphi}{\partial \tau} = -\frac{1-\varepsilon+\tilde{\varepsilon}}{\tilde{\varepsilon}} A\left(1-\varepsilon A \sigma_y^2\right) \tag{24}$$

It can be decomposed into a growth-diminishing distortionary effect which is associated with the reduction in expected capital return and a growth-enhancing insurance effect which is associated with the decline in capital risk. For a detailed discussion of the effects of taxation within stochastic models of endogenous growth see for example Eaton (1981), Turnovsky (1995a), Smith (1996), Corsetti (1997) or Clemens and Soretz (1997). The direct effect of taxation on growth can be shown to be negative if and only if the certainty equivalent of the portfolio return is positive. This condition can be interpreted in the following way. With a positive certainty equivalent, risk does not dominate the model. The technology is 'certain enough' to ensure that the effects of the underlying deterministic structure prevail. A negative certainty equivalent describes a situation where the uncertain capital income flow yields the same utility as a certain interest rate which is negative. The following analysis will be restricted to parameter settings which ensure a positive certainty equivalent, because this situation fits better to reality.

But furthermore, in this model there is an indirect impact of income taxation on growth. Any change in the income tax rate leads to a simultaneous adjustment of portfolio choice and government debt, respectively. Since with the underlying environmental preferences expected growth depends on the portfolio share of capital, the adjustment of portfolio choice induces an ambiguous indirect growth effect which is discussed in the following.

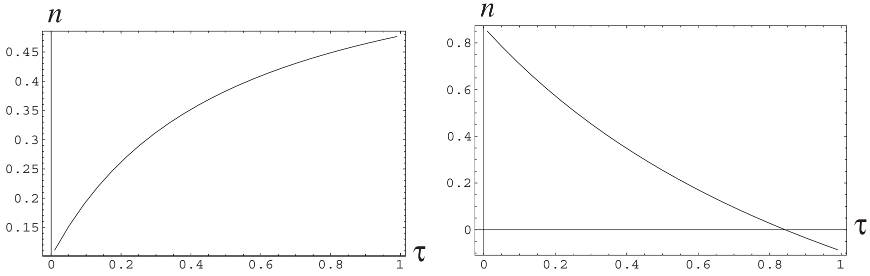
The equilibrium portfolio composition is derived endogenously out of the optimization of households. Note that the portfolio share of capital can be displayed as the ratio between the propensity to consume out of wealth, μ, and the relation of consumption and capital, c/k. Substitution of (16), the market clearing condition $c/k = A - \varphi - \eta$ and the growth rate (17) leads to an expression for the portfolio share n which can be solved as follows:

$$n = \frac{\rho(1-\vartheta) - (\varepsilon-1)\left[\vartheta(A-\eta) - (1-\tau)A + (\varepsilon/2)(1+\vartheta-2\tau)A^2\sigma_y^2\right]}{(1-\vartheta)\left[\varepsilon(A-\eta) - (1-\tau)A + \rho - (\varepsilon/2)(\varepsilon-1+2\tau)A^2\sigma_y^2\right]} \tag{25}$$

η denotes the relation between abatement and capital stock, E/k, and $\vartheta = \alpha\gamma(1-\delta)$ is defined for notational convenience.

The indirect growth effect of income taxation can be split into the influence of portfolio choice on growth, $\partial\varphi/\partial n$, multiplied by the influence of income taxation on portfolio choice, $\partial n/\partial\tau$. Equilibrium growth unambiguously increases with a rise in the portfolio share of physical capital:

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.



(a) (b)

FIG. 1 Portfolio Response to a Tax Increase for Small versus High Relative Risk Aversion: (a) Relative Risk Aversion $\varepsilon = 0.5$; (b) Relative Risk Aversion $\varepsilon = 3$

$$\frac{\partial\varphi}{\partial n} = \frac{1}{\tilde{\varepsilon}}\left[(1-\tau)A\left(1-\varepsilon A\sigma_y^2\right)\right]\frac{(1-\vartheta)\vartheta}{[1+\vartheta(1-n)]^2} > 0 \tag{26}$$

Nevertheless, the effect of income taxation on portfolio composition, $\partial n/\partial\tau$, is ambiguous. On the one hand expected net capital return decreases with a rise in the tax rate. Consequently, physical capital gets less attractive compared with government bonds. On the other hand, the volatility of the capital return diminishes with increasing taxation whereas the uncertainty of the bond return rises.

The effect of income taxation on portfolio composition is illustrated in Fig. 1. The parameters are set in the following way: $\rho = 0.03$, $\gamma = 1$, $A = 0.4$, $\sigma_y = 0.01$, $\delta = 0.5$, $\alpha = 0.5$. η is evaluated according to equation (29) developed below at the optimal level. For small values of relative risk aversion ε the derivative of the portfolio share n with respect to τ can be shown to be positive, whereas with a relative risk aversion sufficiently high the derivative becomes negative. In the first case, the positive effect of taxation on the portfolio share of capital dominates: an increase in the tax rate induces a shift towards physical capital. In contrast, if the agents are sufficiently risk averse, the negative effect of taxation on the portfolio share of capital dominates. In this case, a decrease in expected net capital return induces a decrease in capital demand. The optimal portfolio share of physical capital is reduced. Figure 1(b) additionally shows that there is an upper bound to the income tax rate which must be kept in order to ensure feasible solutions with positive values for capital.

4.2 Optimal Environmental Policy

In the model considered here, optimal fiscal and environmental policy can be decomposed into two steps. First, the government has to choose the optimal

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

level of abatement expenditures. Second, optimal financing has to be analyzed. In order to allow for a steady state, abatement expenditures have to grow at the common equilibrium rate. Hence, government is restricted to choosing a constant ratio η between abatement activities and physical capital. Referring to optimal pollution control, expected lifetime utility (22) is maximized with respect to the environmental expenditure rate η and with respect to the growth rate φ:

$$\frac{\partial U}{\partial \eta} \overset{!}{=} 0 \quad \Leftrightarrow \quad \eta^* = \frac{\alpha\gamma}{1+\alpha\gamma}(A-\varphi) \tag{27}$$

$$\frac{\partial U}{\partial \varphi} \overset{!}{=} 0 \quad \Leftrightarrow \quad \varphi^* = \frac{1}{\varepsilon}(A-\eta-\rho) + \frac{\varepsilon-1}{2}A^2\sigma_y^2 \tag{28}$$

Combination of these two optimality conditions leads to

$$\eta^* = \frac{\alpha\gamma}{\varepsilon(1+\alpha\gamma)-\alpha\gamma}[\rho+(\varepsilon-1)A] + \varepsilon\frac{1-\varepsilon}{2}A^2\sigma_y^2 \tag{29}$$

$$\varphi^* = \frac{1+\alpha\gamma}{\varepsilon(1+\alpha\gamma)-\alpha\gamma}\left(\frac{A}{1+\alpha\gamma} - \rho + \varepsilon\frac{\varepsilon-1}{2}A^2\sigma_y^2\right) \tag{30}$$

What is the Impact of Uncertainty on the Social Optimum? The optimal rate of abatement activities is given by equation (29) and differs with respect to the second term from the corresponding deterministic model. That is, optimal environmental care increases (decreases) with uncertainty if risk aversion is less (higher) than unity. Hence, in general the outcome of the deterministic model does not extend to the case of uncertainty. If risk aversion is sufficiently low ($\varepsilon < 1$), optimal pollution control is underestimated by the setting without risk (and vice versa). Only with logarithmic preferences ($\varepsilon = 1$) is optimal environmental care independent of uncertainty. In this case, the income and the substitution effect of risk on environmental care balance.

These results are due to precautionary savings.[5] Equation (30) indicates that optimal capital accumulation under uncertainty differs from the results derived for the case of certainty if risk aversion deviates from unity. Optimal growth is underestimated (overestimated) by the model without risk for a degree of risk aversion higher (lower) than unity. In general, uncertainty has a positive income and a negative substitution effect on savings. With a relative risk aversion sufficiently high, the income effect prevails and there is a motive for precautionary savings (see Leland, 1968; Sandmo, 1970). In this

---

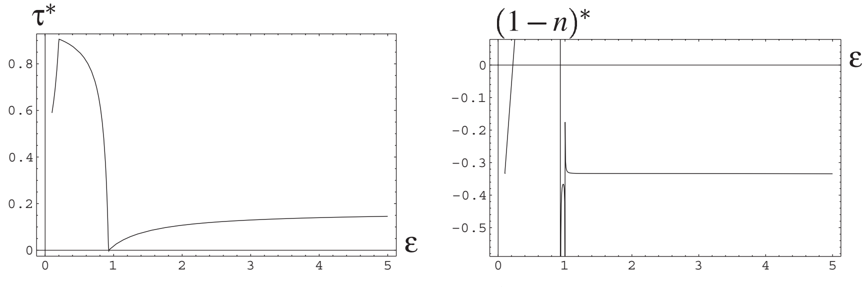
[5] Note that this outcome relies crucially on the assumption of a productivity shock which is a pure capital risk in the terms of Sandmo (1970). For other types of risk, e.g. uncertainty about the damage of pollution, the results may change substantially.

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

case uncertainty leads to higher savings. Instantaneous consumption together with abatement expenditures are reduced. Vice versa, if the coefficient of relative risk aversion is sufficiently low, an increase in uncertainty induces a reduction in optimal accumulation accompanied by a rise in instantaneous consumption as well as environmental expenditures.

How Do Environmental Preferences Affect Optimal Growth? The impact of environmental preferences on optimal accumulation (30) is ambiguous. Again, there are counteracting substitution and income effects, as already analyzed by Smulders and Gradus (1996). On the one hand, stronger environmental preferences induce an increase in instantaneous abatement expenditures. The environmental expenditure ratio rises at the cost of savings (substitution effect). On the other hand, an increase in capital accumulation enhances future possibilities for abatement. Therefore, savings tend to increase (income effect). Nevertheless, it can be shown that the negative substitution effect dominates for feasible solutions with positive consumption.

Determination of Optimal Income Taxation. Within the second step of optimal policy, government solves for the optimal financing of environmental expenditures. Optimal financing requires the identity of decentralized expected growth (17) and optimal expected growth (30). Since income taxation has various and ambiguous impacts on expected growth, optimal financing will only be illustrated numerically. Using the portfolio share of capital (25) as well as optimal environmental care (29) to determine the decentralized growth rate (17) and equating decentralized and optimal growth leads to an optimal tax rate as given by Figs 2(a) and 3(a). Government debt results residually and is illustrated in Figs 2(b) and 3(b). The parameter settings are the same as in Fig. 1.



(a) (b)

FIG. 2 Relative Risk Aversion and Optimal Financing of Environmental Care: (a) Optimal Tax Rate; (b) Optimal Government Debt

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

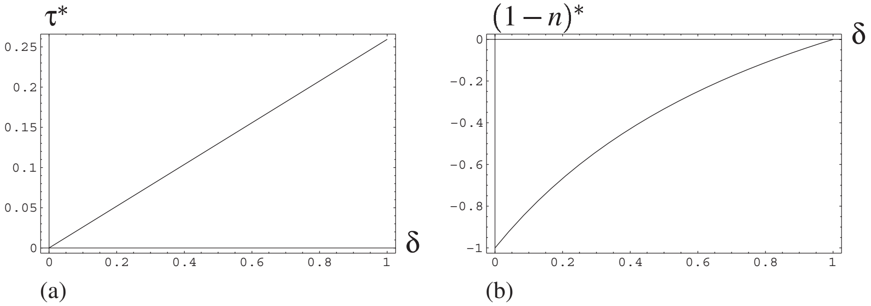


FIG. 3 Perception of Aggregate Capital and Optimal Financing of Environmental Care: (a) Optimal Tax Rate; (b) Optimal Government Debt

What is the Impact of Risk Aversion on Optimal Fiscal Policy? It can be seen that the optimal tax policy crucially depends on the degree of relative risk aversion. Various counteracting effects have to be considered. First, with an increase in relative risk aversion the insurance effect of income taxation gains importance. Hence, the magnitude of the individual reaction on any tax policy increases. Second, the motive for precautionary savings has more weight and changes the growth rate. Third, with the constant relative risk aversion type of preferences considered here, relative risk aversion is the reciprocal of the elasticity of intertemporal substitution. And the deviation of equilibrium from optimal growth is an increasing function of the intertemporal substitution elasticity. The different and in part opposing effects sum up in an optimal tax rate, as illustrated in Fig. 2(a).

Figure 2(b) shows that the optimal portfolio share of government bonds is negative if the degree of relative risk aversion is sufficiently high. Thus, the equilibrium value of government bonds is negative for optimal financing and government becomes a net creditor to the public. Optimal expenditures for environmental care are higher than optimal tax revenues. The negative externality of capital accumulation allows for positive income taxation in order to diminish equilibrium growth, but the revenue out of income taxation is not sufficient for optimal environmental care. The government budget is closed via interest payments of private households.

How Does Optimal Financing Change with Perception? Figure 3 shows the influence of the perception of capital on the optimal financing of abatement activities. Additionally to the previous figures, the degree of relative risk aversion ε is set to 3. An increase in the perception parameter δ corresponds to a decline in the anticipation of the interdependence between individual and aggregate capital. Hence, with increasing δ agents neglect a greater part of their contribution to pollution. The negative externality of capital

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

		φ	η^*	φ^*	τ^*	$(1-n)^*$
$\gamma\uparrow$		↓	↑	↓	?	?
$\delta\uparrow$		↑	—	—	↑	↑
$\sigma\uparrow$	ε high	↑	↓	↑	?	?
	ε low	↓	↑	↓		

FIG. 4 Changes in Preferences, Perception and Uncertainty

accumulation rises with δ and allows for an increase in optimal income taxation. Hence, the optimal value of government bonds is still negative but diminishes in absolute value.

The results derived so far are summarized in Fig. 4. Stronger environmental preferences (higher γ) unambiguously lead to a decrease in equilibrium growth φ. The optimal environmental expenditure rate η^* increases, and optimal expected growth φ^* decreases. The effects of environmental preferences on optimal tax policy τ^*, as well as on optimal government debt $(1 - n)^*$, are complex and depend on the parameter setting.

Perception of individual responsibility for environmental degradation does not influence optimal growth and environmental care. Nevertheless, equilibrium growth increases with decreasing environmental responsibility (increasing δ). As shown in Fig. 3, there is more need for internalizing income taxation. With increasing tax revenues, the value of government bonds increases.

The impact of uncertainty depends mainly on the degree of relative risk aversion. When risk aversion is sufficiently high (low), a rise in uncertainty leads to an increase (decrease) in equilibrium and optimal growth due to precautionary savings. Simultaneously, environmental expenditures decrease (increase) together with instantaneous consumption. Nevertheless, the impact of uncertainty on optimal financing of environmental expenditures again depends on additional relations between the preference and technology parameters.

5 A SPECIAL CASE: AGENTS COMPLETELY NEGLECT THEIR CONTRIBUTION TO AGGREGATE CAPITAL

In this section I demonstrate the polar case where individuals completely ignore their influence on aggregate capital and hence on pollution. This corresponds to the setting $\delta = 1$. Since the representative agent does not take into account the influence of capital accumulation on pollution, portfolio composition does not have welfare effects in this situation. The value of government bonds enters neither instantaneous utility nor the value function,

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

because the development of the real sector is not affected by government debt. Hence, equilibrium growth is independent of the portfolio share of physical capital:

$$\varphi_1 = \frac{1}{\varepsilon}[(1-\tau)A-\rho]+\frac{1}{2}(\varepsilon-1+2\tau)A^2\sigma_y^2 \tag{31}$$

In this setting the environment has no direct impact on equilibrium growth. Agents ignore their individual influence on aggregate capital as well as on aggregate abatement expenditures. Thus, pollution is perceived to be purely exogenous and does not have any consequences for individual optimization. Nevertheless, the government has to take environmental degradation as well as abatement activities into account in order to set the fiscal policy parameters optimally. The tax rate on deterministic income components as well as the taxation of uncertain income parts affect savings. Thus, accumulation is adjusted optimally through income taxation. More simply, optimal taxation balances marginal expected utility out of consumption for marginal expected disutility out of pollution.

The optimal ratio of abatement effort and capital η^* is independent of the perception of capital and still given by equation (29). But now it is possible to evaluate a closed-form solution for optimal income taxation. Equating decentralized growth φ_1 and optimal growth φ^* now leads to

$$\tau_1^* = \frac{\alpha\gamma}{A[\varepsilon+\alpha\gamma(1-\varepsilon)](1-\varepsilon A\sigma_y^2)}\left[\rho+(\varepsilon-1)A-\varepsilon\frac{\varepsilon-1}{2}A^2\sigma_y^2\right] \tag{32}$$

The effect of uncertainty on the optimal tax rate is ambiguous and depends on the degree of risk aversion. If relative risk aversion is higher than unity ($\varepsilon > 1$), there is a motive for precautionary savings. The reduction of risk associated with the taxation of stochastic income parts discourages accumulation and encourages consumption. Thus, the growth-diminishing effect of taxation of deterministic income components (due to a reduction in expected capital return) is reinforced by the negative growth effect of taxation of uncertain income parts. For this reason, the optimal tax rate tends to decrease with risk in a setting with strong risk aversion. Nevertheless, there are ambiguous additional effects through government debt and portfolio decision. Thus, the overall effect of uncertainty on optimal taxation is ambiguous.

If instead relative risk aversion is less than unity, agents do not have a motive for precautionary savings. The substitution effect of uncertainty on savings dominates and individuals reduce savings when uncertainty increases. Taxation of stochastic income parts reduces the volatility of capital return and thus induces a decrease in the risk associated with capital accumulation. With a relative risk aversion less than unity individuals now increase savings because, due to the decrease in risk, capital accumulation gets more attractive. Hence, the growth effects of income taxation of deterministic and

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

stochastic income components are counteracting and the overall growth effect of income taxation is smaller than under certainty.[6] This leads immediately to an optimal income tax rate which is increased by uncertainty.

Furthermore, it is possible to evaluate the equilibrium portfolio share of physical capital:

$$n_1 = \frac{\rho + (\varepsilon - 1)\left(\varphi_1 - \frac{1}{2}\varepsilon A^2 \sigma_y^2\right)}{A - \eta - \varphi_1} \tag{33}$$

Tax policy influences expected growth and thus portfolio composition. It can be shown that the portfolio share diminishes with an increase in the income tax rate:

$$\frac{\partial n_1}{\partial \tau} = -\frac{A\left(1 - \varepsilon A \sigma_y^2\right)}{A - \eta - \varphi_1} < 0 \tag{34}$$

Taxation of the capital return reduces the incentive for capital accumulation and induces a shift towards government bonds. The tax revenues rise and induce *ceteris paribus* (with constant environmental expenditure) an increase in the value of government bonds.

For optimal environmental care η^* and optimal income tax τ^*, equation (33) leads to a portfolio share of $n_1 = 1$. That is, abatement expenditures are completely financed by the income tax. In this respect, the outcomes of the deterministic setting continue to hold. The optimal tax rate which completely internalizes the external effect simultaneously collects the right amount to finance abatement. Thus, there is no need for further financing via growth-neutral government debt.

The outcomes of this polar case ($\delta = 1$) are summarized in Fig. 5. The line with negative slope (as developed in equation (34)) identifies the correspondence between the two financing instruments government debt and income tax given in equation (33). Hence, this line indicates all feasible fiscal policies. Optimal tax policy is given by τ^* and $n = 1$. If the tax rate is less than τ^*, the portfolio share of capital exceeds unity. This situation corresponds to the case with negative value of government bonds. Government becomes a net creditor to the public.

6 CONCLUSION

In this paper a stochastic endogenous growth model with pollution is analyzed. Pollution causes disutility and depends on the ratio between abatement

---

[6] As long as the certainty equivalent of the portfolio return is positive, the growth-diminishing effect of the taxation of deterministic income parts dominates. As noted above, I restrict the analysis to parameter settings which ensure a positive certainty equivalent.

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

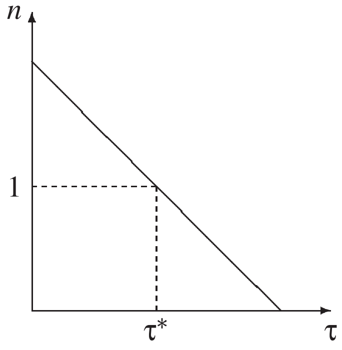


FIG. 5 Optimal Fiscal Policy for $\delta = 1$

activities and the capital stock. Since individuals neglect their contribution to aggregate environmental expenditures, these are financed by income taxation and government debt. Furthermore, the agents feel only partially responsible for the influence of capital on pollution. Perception of aggregate capital is parameterized and both polar cases are included: the setting where individuals completely ignore their contribution to aggregate capital accumulation and the contrary case with perfect information about this relation.

Equilibrium growth is analyzed and it is shown that stronger environmental preferences induce a negative growth effect, as the costs of accumulation increase. Furthermore, equilibrium growth depends on the individual's perception of aggregate capital. With an increase in the part of privately perceived influence on pollution, the negative impact of capital accumulation is given more weight within individual optimization. Hence, expected growth decreases.

Fiscal policy as well as optimal environmental care are analyzed. Optimal abatement activities are shown to depend on risk in an ambiguous way. If relative risk aversion is sufficiently high (low), the optimal environmental expenditure rate decreases (increases) with uncertainty. Thus, the outcomes of the deterministic setting in general do not apply to the case of uncertainty. Only if the degree of relative risk aversion is unity do the results remain unchanged.

As long as the agents perceive a dependence between individual and aggregate capital, there is an additional negative effect of capital accumulation on intertemporal utility. This leads to an interdependence between government debt and expected growth. Hence, in contrast to most endogenous growth models, income taxation not only has a direct effect on expected growth, but it also influences portfolio choice, and this leads to an additional indirect impact on the growth rate. Due to the different counteracting growth

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

effects of income taxation, numerical simulation of optimal fiscal policy is required. It is shown that the optimal income tax rate depends on the relation between relative risk aversion, perception of capital and environmental preferences.

In the last section, the outcomes are evaluated in the special case where the agents perceive aggregate capital as completely exogenous. It is shown that the portfolio share of physical capital decreases with the income tax rate and that optimal taxation of environmental expenditures in this case requires complete income tax financing.

References

Ayong Le Kama, A. (2001). 'Preservation and Exogenous Uncertain Preferences', *Economic Theory*, Vol. 18, pp. 745–752.

Baranzini, A. and Bourguignon, F. (1995). 'Is Sustainable Growth Optimal?', *International Tax and Public Finance*, Vol. 2, pp. 341–356.

Beltratti, A., Chichilnisky, G. and Heal, G. M. (1998). 'Uncertain Future Preferences and Conservation', in G. Chichilnisky, G. M. Heal and A. Vercelli (eds), *Sustainability: Dynamics and Uncertainty*, Dordrecht, Kluwer Academic, pp. 257–275.

Brock, W. A. and Mirman, L. J. (1972). 'Optimal Economic Growth and Uncertainty: the Discounted Case', *Journal of Economic Theory*, Vol. 4, pp. 479–513.

Brock, W. A. and Mirman, L. J. (1973). 'Optimal Economic Growth and Uncertainty: the No-discounting Case', *International Economic Review*, Vol. 14, pp. 560–573.

Chichilnisky, G. and Heal, G. M. (1998). 'Financial Markets for Unknown Risks', in G. Chichilnisky, G. M. Heal and A. Vercelli (eds), *Sustainability: Dynamics and Uncertainty*, Dordrecht, Kluwer Academic, pp. 277–294.

Clarke, H. R. and Reed, W. J. (1994). 'Consumption/Pollution Tradeoffs in an Environment Vulnerable to Pollution-related Catastrophic Collapse', *Journal of Economic Dynamics and Control*, Vol. 18, pp. 991–1010.

Clemens, C. and Soretz, S. (1997). 'Macroeconomic Effects of Income Taxation in a Model of Stochastic Growth', *Finanzarchiv, N. F.*, Vol. 54, pp. 471–493.

Clemens, C. and Soretz, S. (1999). 'Konsequenzen des Zins- und Einkommensrisikos auf das wirtschaftliche Wachstum', *Zeitschrift für Wirtschafts- und Sozialwissenschaften*, Vol. 119, pp. 593–614.

Corsetti, G. (1997). 'A Portfolio Approach to Endogenous Growth: Equilibrium and Optimal Policy', *Journal of Economic Dynamics and Control*, Vol. 21, pp. 1627–1644.

Devereux, M. B. and Smith, G. W. (1994). 'International Risk Sharing and Economic Growth', *International Economic Review*, Vol. 35, pp. 535–550.

Eaton, J. (1981). 'Fiscal Policy, Inflation and the Accumulation of Risky Capital', *Review of Economic Studies*, Vol. 48, pp. 435–445.

Edwards, J. H. Y. (1990). 'Congestion Function Specification and the "Publicness" of Local Public Goods', *Journal of Urban Economics*, Vol. 27, pp. 80–96.

Forster, B. (1973). 'Optimal Capital Accumulation in a Polluted Environment', *Southern Economic Journal*, Vol. 39, pp. 544–547.

Glomm, G. and Ravikumar, B. (1994). 'Public Investment in Infrastructure in a Simple Growth Model', *Journal of Economic Dynamics and Control*, Vol. 18, pp. 1173–1187.

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

Gradus, R. and Smulders, S. (1993). ‘The Trade-off Between Environmental Care and Long-term Growth—Pollution in Three Prototype Growth Models’, *Journal of Economics*, Vol. 58, pp. 25–51.

Greenwood, J. and Jovanovic, B. (1990). ‘Financial Development, Growth, and the Distribution of Income’, *Journal of Political Economy*, Vol. 98, pp. 1076–1107.

Grossman, G. M. and Krueger, A. B. (1995). ‘Economic Growth and the Environment’, *Quarterly Journal of Economics*, Vol. 110, pp. 353–377.

Gruver, G. (1976). ‘Optimal Investment and Pollution Control in a Neoclassical Growth Context’, *Journal of Environmental Economics and Management*, Vol. 5, pp. 165–177.

de Hek, P. A. (1999). ‘On Endogenous Growth under Uncertainty’, *International Economic Review*, Vol. 40, pp. 727–744.

Jones, L. E., Manuelli, R. E. and Stacchetti, E. (1999). ‘Technology (and Policy) Shocks in Models of Endogenous Growth’, *NBER Working Paper 7063*, National Bureau of Economic Research, Cambridge, MA.

Leland, H. E. (1968). ‘Saving and Uncertainty: the Precautionary Demand for Saving’, *Quarterly Journal of Economics*, Vol. 82, pp. 465–473.

Leland, H. E. (1974). ‘Optimal Growth in a Stochastic Environment’, *Review of Economic Studies*, Vol. 41, pp. 75–86.

Levhari, D. and Srinivasan, T. N. (1969). ‘Optimal Savings under Uncertainty’, *Review of Economic Studies*, Vol. 36, pp. 153–163.

Lucas, R. E. (1988). ‘On the Mechanics of Economic Development’, *Journal of Monetary Economics*, Vol. 22, pp. 3–42.

Luptacik, M. and Schubert, U. (1982). *Optimal Economic Growth and the Environment, Economic Theory of Natural Resources*, Wien, Physica.

Malliaris, A. G. and Brock, W. A. (1982). *Stochastic Methods in Economics and Finance*, Amsterdam, North-Holland.

Merton, R. C. (1992). *Continuous-time Finance*, Cambridge, MA, Blackwell.

Obstfeld, M. (1994). ‘Risk-taking, Global Diversification, and Growth’, *American Economic Review*, Vol. 84, pp. 1310–1329.

Rebelo, S. (1991). ‘Long-run Policy Analysis and Long-run Growth’, *Journal of Political Economy*, Vol. 99, pp. 500–521.

Reis, A. B. (2001). ‘Endogenous Growth and the Possibility of Eliminating Pollution’, *Journal of Environmental Economics and Management*, Vol. 42, pp. 360–373.

Sandmo, A. (1970). ‘The Effect of Uncertainty on Savings Decisions’, *Review of Economic Studies*, Vol. 37, pp. 353–360.

Siebert, H. (1987). *Economics of the Environment*, Berlin, Springer.

Smith, W. T. (1996). ‘Feasibility and Transversality Conditions for Models of Portfolio Choice with Non-expected Utility in Continuous Time’, *Economics Letters*, Vol. 53, pp. 123–131.

Smulders, S. and Gradus, R. (1996). ‘Pollution Abatement and Long-term Growth’, *European Journal of Political Economy*, Vol. 12, pp. 505–532.

Stokey, N. L. (1998). ‘Are There Limits to Growth?’, *International Economic Review*, Vol. 39, pp. 1–31.

Turnovsky, S. J. (1993). ‘Macroeconomic Policies, Growth, and Welfare in a Stochastic Economy’, *International Economic Review*, Vol. 34, pp. 953–981.

Turnovsky, S. J. (1995a). ‘Optimal Tax Policy in a Stochastically Growing Economy’, *Japanese Economic Review*, Vol. 46, pp. 125–147.

Turnovsky, S. J. (1995b). *Methods of Macroeconomic Dynamics*, Cambridge, MA, MIT Press.

Turnovsky, S. J. (1999). ‘On the Role of Government in a Stochastically Growing Economy’, *Journal of Economic Dynamics ond Control*, Vol. 104, pp. 275–298.

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.

Turnovsky, S. J. (2000). 'Fiscal Policy, Elastic Labor Supply, and Endogenous Growth', *Journal of Monetary Economics*, Vol. 45, pp. 185–210.

Turnovsky, S. J. and Fisher, W. H. (1998). 'Public Investment, Congestion, and Private Capital Accumulation', *Economic Journal*, Vol. 108, pp. 399–413.

Van der Ploeg, F. and Withagen, C. (1991). 'Pollution Control and the Ramsey Problem', *Environmental and Resource Economics*, Vol. 1, pp. 215–230.

© Blackwell Publishing Ltd and The Victoria University of Manchester, 2003.